



# Application of Artificial Intelligence Models to Estimate Discharge over Semicircular Weirs

Majid Niazkar <sup>1</sup>, Nasser Talebbeydokhti <sup>2\*</sup>, Seied Hosein Afzali <sup>3</sup>

<sup>1</sup>Ph.D. of Civil Engineering, Department of Civil and Environmental Engineering, Shiraz University, Shiraz, Iran

<sup>2\*</sup> Professor, Department of Civil and Environmental Engineering, Shiraz University, Shiraz, Iran  
(taleb@shirazu.ac.ir)

<sup>3</sup> Associate Professor, Department of Civil and Environmental Engineering, Shiraz University, Shiraz, Iran

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## ABSTRACT

Weirs are one of the widely used hydraulic structures for measuring discharge in open channels. This study applies two artificial intelligence models named artificial neural network (ANN) and genetic programming (GP) to predict discharge flowing over semicircular weirs with different openings including sharp and semicircular crests. The considered data base was selected from the literature. The results of AI models were compared with those of two empirical formulas, which have been developed based on the same data for this purpose. Four evaluation criteria were considered for comparing the estimated discharges. The results obviously indicate that GP outperforms others based on the considered criteria.

## Keywords:

Semicircular weirs, discharge artificial intelligence models, artificial neural network, genetic programming.

## 1. Introduction

Weirs are commonly utilized as one of the most conventional devices for flow measurement in open channels. They are generally classified based on the shape of crests and their opening types. The most common type of weirs may be the sharp-crested ones, which has either a V-notch or rectangular opening while trapezoidal weirs, semicircular weirs, triangular weirs, compound weirs, and flat-topped weirs are some other types of weirs. Different crest geometry or opening shapes bring about different characteristics to each type of weir while this variety made each weir be suitable for a specific applications in practice.



Although numerous studies have been conducted on weirs with sharp crests, only a few ones focused on circular sharp-crested ones [1-4]. Commonly, a circular-shape weir is a circular control section, which is placed in a vertical thin plate. This plate is normally located laterally in a straight open channel [5]. From practical point of view, circular weirs may be beneficial as their crest can be turned and beveled with precision while they do not have to be leveled. In order to avoid of developing a nappe, circular sharp-crested weirs are practically built fully contracted while this arrangement provides a suitable distance between the control section and the bed and sides of the approach channel [2, 4].

The influences of crest configuration or opening shape on characteristics of flow passing over either normal or oblique weirs have been widely investigated. As an example, Greve studied flow characteristics over circular, parabolic, and triangular weirs [5]. Stevens developed a relatively complex relation between water head and the theoretical discharge [2]. Balachandar et al. [1] proposed a simple procedure to determine coefficients of a model for estimating discharge of a sharp-crested circular weir. They took into account channel width and crest height above the channel bed to derive coefficients of their discharge model while experimental data was utilized to verify the discharge model. A simplified equation was also recommended for estimating coefficient of the discharge equation using a field database [1].

Chunrong et al. [6] studied the hydraulics of the circular-crested weirs from both numerical and experimental points of view. The former solved the Reynolds-averaged Navier-Stokes equations to determine a mean flow field using the  $k-\epsilon$  turbulent model. The obtained results from the numerical model agreed with experimental data. Moreover, water flowing over a semicircular weir was examined by Castro Orgaz et al. [7]. They used simple equations incorporating streamline curvature effects and compared their discharge estimations with experimental data. By utilizing experimental data, Vatankhah [4] suggested a theoretical equation for sharp-crested weirs with circular opening, which may be applied to predict actual discharge.

Irzooki et al. [8] experimentally investigated the flow characteristics passing over semicircular weirs with both sharp and semicircular crests. They exploited four different values of weir openings and three crest heights. Based on their experiment data, they suggested one equation for sharp crests and one equation for semicircular crests for estimating actual flow rate passing over semicircular weirs [8]. Niazkar and Afzali [5] proposed simple equations to improve estimation results when discharge is measured with semi-circular weirs with sharp and semi-circular crests. The recommended relations were developed by using a hybrid method based on an experimental database. The proposed relations were compared with the ones available in the literature. They found that the recommended hybrid method not only was successful in calibrating process but also enabled prediction of discharge measured by semicircular weirs with more accuracy than other models.

Artificial intelligence (AI) models have been successfully used to develop discharge models for different types of weirs. Khorchani and Blanpain [9] utilized Artificial Neural Network (ANN) to present a discharge model for side weirs. The suggested technique provided the opportunity of considering both geometrical and hydraulic characteristics of the overflow structure. Emiroglu et al. [10] assessed ANN for estimating discharge capacity of a triangular labyrinth side-weir. The discharge coefficient was determined using 2500 laboratory data. The results obtained by ANN were compared with those of multi nonlinear regression models. Three comparisons performance



evaluation criteria including root mean square errors (RMSE), mean absolute errors (MAE), and correlation coefficient were used for comparison purpose. According to this comparison, ANN was identified as an appropriate method for developing discharge models. Salmasi et al. [11] conducted a series of laboratory experiments to explore two potential effects including (1) the impact of crest length on flow direction and (2) the impact of the step height of broad-crested rectangular weir on discharge coefficient. The achieved results were compared with those of genetic programming (GP) and ANN models to appraise the applicability of these techniques in determination of discharge coefficients. They claimed that the comparison of results indicated that GP is more efficient than ANN for estimating discharge coefficients. Although ANN and GP have shown successful performance in predicting discharge over several kinds of weirs, they have not been applied to estimate discharge for semi-circular weirs based on the current literature.

In the present study, authentic experimental data were used to obtain superb prediction models for both sharp-crested and semi-circular-crested weirs with semi-circular openings. Both ANN and GP, as two powerful AI models, were utilized to estimate discharge flowing over semi-circular weirs. The comparison of the obtained results and those of previous formulas using statistical criteria like RMSE, mean absolute relative error (MARE), and MAE demonstrates that AI models perform better than the ones available in the literature.

## 2. Methodology

### 2.1. Dimensional analysis for flow over semi-circular weirs

Based on dimensional analysis conducted in previous studies [5, 8], the parameters, which affect the discharge passing over the weir with a semi-circular opening are shown in Eq. 1:

$$Q = f(H_0, R, P, g, \rho, \mu) \quad (1)$$

Where  $Q$  is discharge [ $L^3T^{-1}$ ],  $f$  is a function,  $R$  is the radius of the weir opening [L],  $P$  is the height of the weir crest [L],  $H_0$  is flow depth over the crest [L],  $g$  is the gravity acceleration [ $LT^{-2}$ ],  $\rho$  is water density [ $ML^{-3}$ ], and  $\mu$  is the dynamic viscosity of water [ $ML^{-1}T^{-1}$ ]. Based on the parameters involved, the following dimensionless parameters may be deduced by applying  $P_i$  theorem:

$$\frac{Q}{H_0^{2.5} g^{0.5}} = f\left(\frac{R}{H_0}, \frac{P}{H_0}, Re\right) \quad (2)$$

Where  $Re$  is Reynolds number. By neglecting the impact of  $Re$ , the dimensionless relation shown in Eq. 3 may be obtained for representing flow over weirs [5, 8]:

$$\frac{Q}{H_0^{2.5} g^{0.5}} = f\left(\frac{R}{H_0}, \frac{P}{H_0}\right) \quad (3)$$



## 2.2. Data base

The data base considered in this study was adopted from the literature [8]. This reliable dataset, which has been utilized for developing relations for estimating discharge in previous studies [5, 8], consists of total 144 data for two semicircular weirs (72 data for sharp crest and 72 data for semicircular crest). These data were measured in a rectangular flume with 16 m long and 0.6 m width. The corresponding experiment was conducted for six different discharge values and four radii of the openings (0.1575, 0.1775, 0.205, and 0.256 m). The range of variation  $\frac{Q}{H_0^{2.5} g^{0.5}}$  with  $\frac{P}{H_0}$  and  $\frac{R}{H_0}$  data are depicted in Figures 1-2 for sharp and semicircular crests, respectively. As shown,  $\frac{Q}{H_0^{2.5} g^{0.5}}$  generally increase with the increase of  $\frac{P}{H_0}$  and  $\frac{R}{H_0}$  for semicircular weirs regardless of the type of the opening.

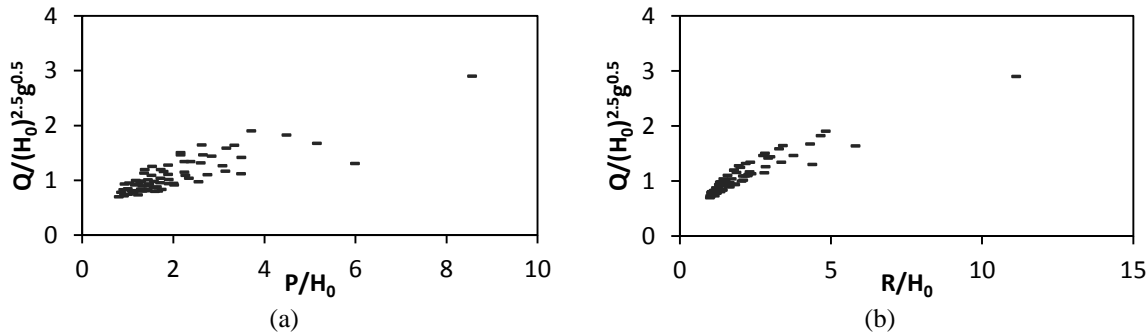


Figure 1. Variation of  $\frac{Q}{H_0^{2.5} g^{0.5}}$  with (a)  $\frac{P}{H_0}$  and (b)  $\frac{R}{H_0}$  for semicircular weir with sharp crest

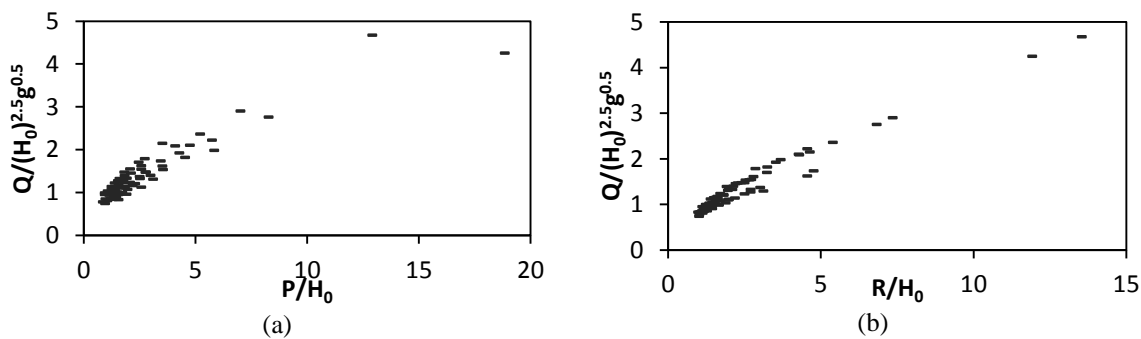


Figure 2. Variation of  $\frac{Q}{H_0^{2.5} g^{0.5}}$  with (a)  $\frac{P}{H_0}$  and (b)  $\frac{R}{H_0}$  for semicircular weir with semicircular crest



### 2.3. Empirical formulas for semi-circular weirs

Irzooki et al. [8] derived two equations for measuring the discharge through sharp- and semi-circular-crested using the semi-circular weirs, as indicated in Eqs. 3-4, respectively:

$$Q = 0.762\sqrt{g}H_0^{1.796}P^{0.183}R^{0.521} \quad (4)$$

$$Q = 0.791\sqrt{g}H_0^{1.779}P^{0.216}R^{0.505} \quad (5)$$

Niazkar and Afzali [5] proposed new accuracy-improved equations by using MHBMO-GRG hybrid method, which has been successfully applied to some problems in water resources engineering field [12-15]. Their equations recommended for calculating discharge from sharp- and semi-circular-crested semi-circular weirs, which are shown in Eqs. 6-7, respectively, yield more precise results than the previous ones:

$$Q = 1.835\sqrt{g}H_0^{2.048}P^{0.242}R^{0.713} \quad (6)$$

$$Q = 1.797\sqrt{g}H_0^{1.954}P^{0.470}R^{0.538} \quad (7)$$

### 2.4. Artificial neural network

Artificial neural networks (ANN), which was originally inspired by the human biological neural network, is one of the most widely used AI models. The similarity between ANN and a human brain may be conceptualized from two different aspects. First, the process of grasp of knowledge is obtained from the environment in both ANN and a human brain. Second, the process of storing the knowledge is conducted using interconnected elements named as neurons. These two similar assets provide an AI technique, which aims not only to capture the relation between different parameters involved in a phenomenon but also to estimate the performance of the phenomenon under other unseen circumstances. The structure of network in an ANN model significantly affects the estimation results. In essence, a typical ANN is consist of three different types of layers. The first layer is the input layer while it neurons contains the input data. This layer interact with the second layer named hidden layer. A neural network basically consists of either one or more hidden layers, whose computation nodes are invariantly called hidden neurons. Commonly, these neurons tends to make a useful relation between the input and output. The more the number of hidden layers, the more the network can extract the third or higher power from its input [16]. However, there is indeed a trade-off in selection of the number of hidden layers as more number of hidden layers may bring about not only an inevitable time-consuming calibration process but also over fitted network. The hidden layers are connected in a way that the output signals of the first hidden layer are utilized as an input to the second hidden layer while this interconnection may continue when more number of hidden layers are considered in the network. The third type of layer is the output layer, whose signals of the neurons are the prediction of ANN to the data inserted in the input layer. Finally, this flexible structure of ANN provides an opportunity to be successfully utilized in various problems in the field of civil and water engineering [9, 12, and 17]. In this study, the ANN was modeled in MATLAB, which is powerful software for implementing numerical solution in water resources engineering field [18-20]. Ten hidden layers were used in the model which was determined by trial



and errors. The other characteristics considered in modeling ANN in this study is as the same as previous studies [13].

## 2.5. Genetic programming

Genetic programming (GP) is an AI model that exploits an evolutionary optimization algorithm, invariantly called genetic algorithm, to determine unknown relations between different variables involved in a typical problem. In GP, programs are presented as syntax trees, where the constants and variables are the terminals and the operators are functions. Generally, it is consisted of five different steps including (1) Terminal set, (2) Function set, (3) Fitness function, (4) Parameters of GP, and (5) Termination and Solution Designation [21-22].

In GP systems, an initial population is first generated by using the available primitives. Afterwards, the fitness of each program is computed while some programs with high fitness may be selected to contribute in genetic operations. Then, new individual programs are created by using genetic operations including cross over and mutation [23]. These three steps are repeated until an acceptable solution is found or some other stopping condition is met. In this study, Discipulus software that has been used for modeling GP in several previous studies [12, 17] was utilized. Also, MSE was used as the fitness function in this application.

## 3. Results and discussion

### 3.1. Performance evaluation criteria

In order to compare the estimated  $Q$  with the corresponding measured values, four performance evaluation criteria were used [5, 14, 17, 22-23]. These criteria are (1) RMSE, (2) MARE, (3) determination coefficient ( $R^2$ ), and (4) relative error ( $RE_i$ ). These criteria are shown in Eqs. 8-12, respectively:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (Q_{estimated} - Q_{measured})^2} \quad (8)$$

$$MARE = \frac{1}{N} \sum_{i=1}^N \frac{|Q_{estimated} - Q_{measured}|}{Q_{measured}} \quad (9)$$

$$R^2 = \left( \frac{\sum_{i=1}^N [(Q_{measured} - \frac{\sum_{i=1}^N Q_{measured}}{N})(Q_{estimated} - \frac{\sum_{i=1}^N Q_{estimated}}{N})]}{\sqrt{\sum_{i=1}^N [(Q_{measured} - \frac{\sum_{i=1}^N Q_{measured}}{N})^2 (Q_{estimated} - \frac{\sum_{i=1}^N Q_{estimated}}{N})^2]}} \right)^2 \quad (10)$$

$$RE_i = \frac{Q_{estimated} - Q_{measured}}{Q_{measured}} \quad \text{for } i = 1, \dots, N \quad (11)$$

Where  $Q_{estimated}$  and  $Q_{measured}$  are the estimated  $Q$  and observed  $Q$ , respectively,  $i$  is the counter, and  $N$  is the number of data.



### 3.2. Results of semicircular weir with sharp crest

In application of artificial intelligence (AI) models (ANN and GP),  $R$ ,  $P$ , and  $H_0$  were used as the input data while  $Q$  was set as the output data. Furthermore, the data base for each opening of semicircular weir was randomly divided into two parts: train (44.44% of total data for train and 22.22% of total data for validation) and test (33.33% of total data) data while the dividing procedure conducted was adopted from the literature [12]. As the name of each data part indicates, the former was used to train and validate the AI models while the latter was utilized to test the performance of different models in predicting discharges.

The performance of the considered AI models is compared in Tables 1-2 with that of the empirical ones available in the literature for semicircular weir with sharp crest for the train and test data, respectively. As shown, the AI models improve MARE, RMSE, and  $R^2$  values of the empirical relations for the train data while GP outperforms others based on Table 1. Additionally, Table 2 shows that GP achieved the best MARE value while Eq. 6 outperformed others in favor of RMSE and  $R^2$ . Furthermore, the AI models obtained better  $R^2$  values than Eq. 4 while GP yield to better RMSE than Eq. 4.

**Table 1: Comparison of the performance of different models for estimating discharge using sharp crest semicircular weir for the train data**

Models	MARE	RMSE	$R^2$
Eq. 4 [8]	0.166	1.268	0.979
Eq. 6 [5]	0.037	0.537	0.993
ANN (this study)	0.022	0.353	0.997
GP (this study)	0.018	0.173	0.999

**Table 2: Comparison of the performance of different models for estimating discharge using sharp crest semicircular weir for the test data**

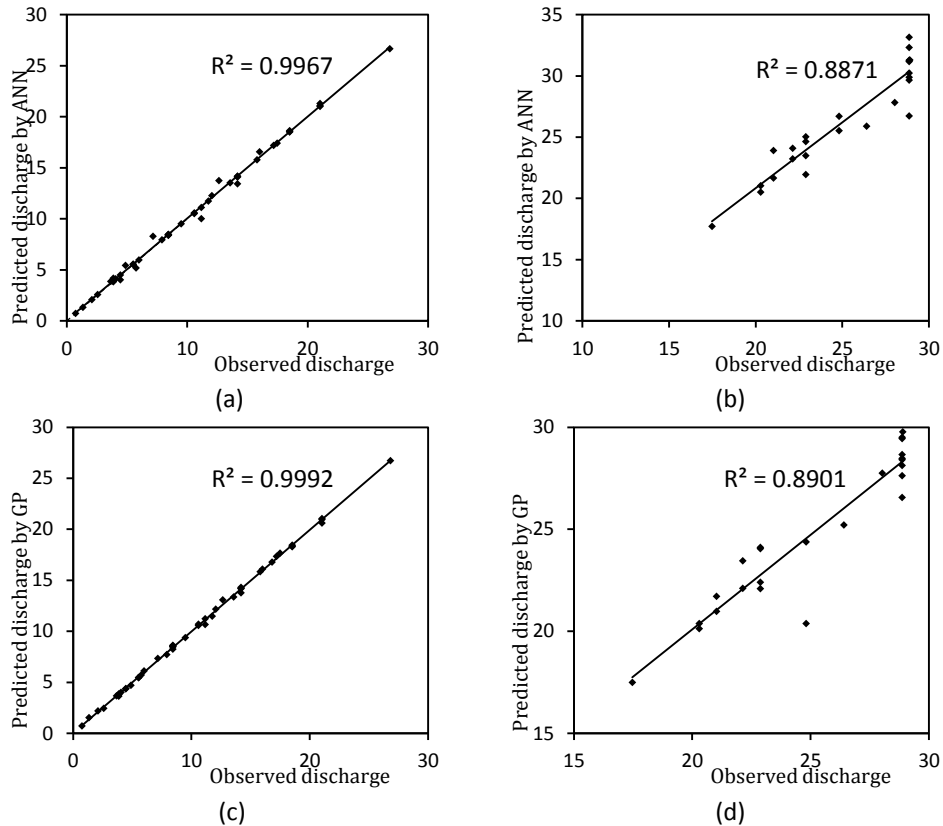
Models	MARE	RMSE	$R^2$
Eq. 4 [8]	0.057	1.472	0.819
Eq. 6 [5]	0.038	1.197	0.918
ANN (this study)	0.060	1.860	0.887
GP (this study)	0.032	1.235	0.890

Figure 3 depicts  $Q_{estimated}$  achieved by the AI models vs.  $Q_{measured}$  for the train and test data while the unit of discharge shown in this figure is liter per second. As shown, GP predicted discharge values better than ANN for both parts of data because GP obtained larger  $R^2$  than ANN.

The variations of  $RE_i$  of discharges predicted by the AI models are plotted for the train and test data in Figure 4. As shown, the range of  $RE_i$  values of ANN is between -0.1 and 0.15 for the train data and between -0.075 and 0.15 for the test data. Moreover, the corresponding bound for GP varies from -0.05 to 0.15 and from -0.18 to 0.06 for the train and test data, respectively. Based on these ranges of  $RE_i$  values, GP performs better than ANN for estimating the discharge of the train data while the minimum and maximum  $RE_i$  values were achieved by GP and ANN for the test

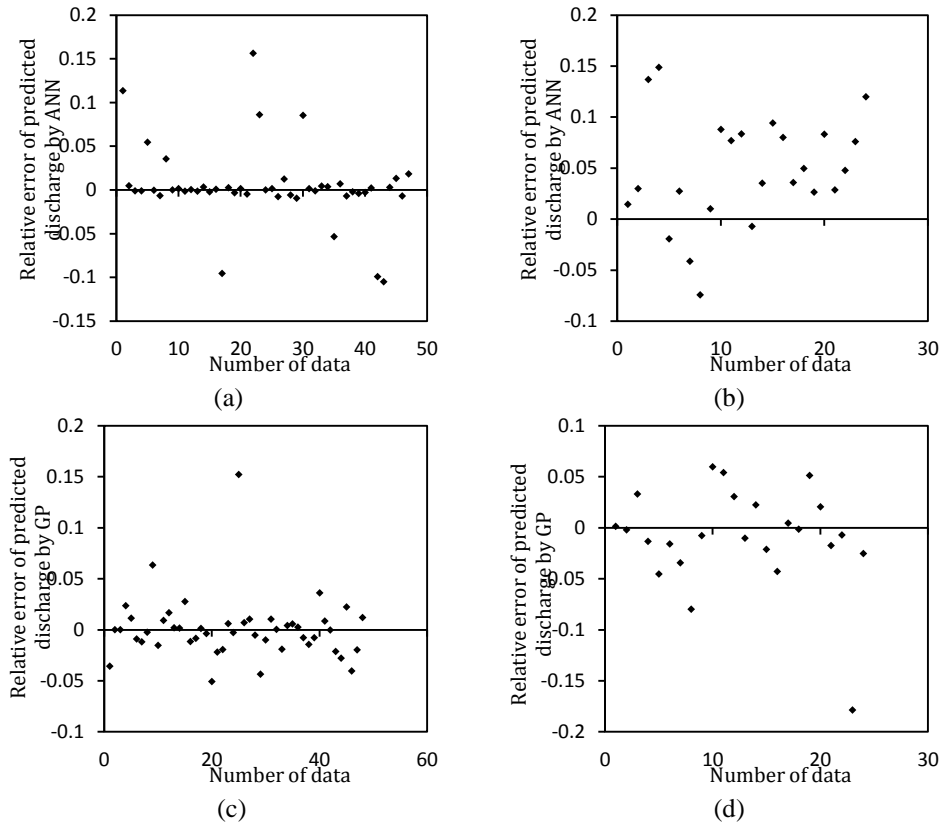


data, respectively. According to the definition of  $RE_i$  presented in Eq. 11, positive  $RE_i$  indicates overestimation while negative  $RE_i$  shows underestimation. Based on Figure 4, GP underestimated the discharge values for most of the test data while ANN overestimated them for the same data.



**Figure 3: Comparison the performance of AI models for predicting discharge for sharp crest semicircular weir: (a) ANN for train data, (b) ANN for test data, (c) GP for train data, and (d) GP for test data.**





**Figure 4. Relative error of predicted discharge for sharp crest semicircular weir: (a) ANN for train data, (b) ANN for test data, (c) GP for train data, and (d) GP for test data**

### 3.3. Results of semicircular weir with semicircular crest

The performances of GP, ANN, and empirical formulas for estimating discharge of semicircular weir with semicircular crest are compared in Tables 3-4 for the train and test data, respectively. Tables 3-4 obviously show the superiority of GP over other models in light of MARE, RMSE, and  $R^2$  criteria for both train and test data. Also, Tables 3-4 indicates that ANN achieved better RMSE and  $R^2$  values in comparison with empirical formulas while they obtained better MARE values than ANN for the train and test data.

**Table 3: Comparison of the performance of different models for estimating discharge using semicircular weir with semicircular crest for the train data**

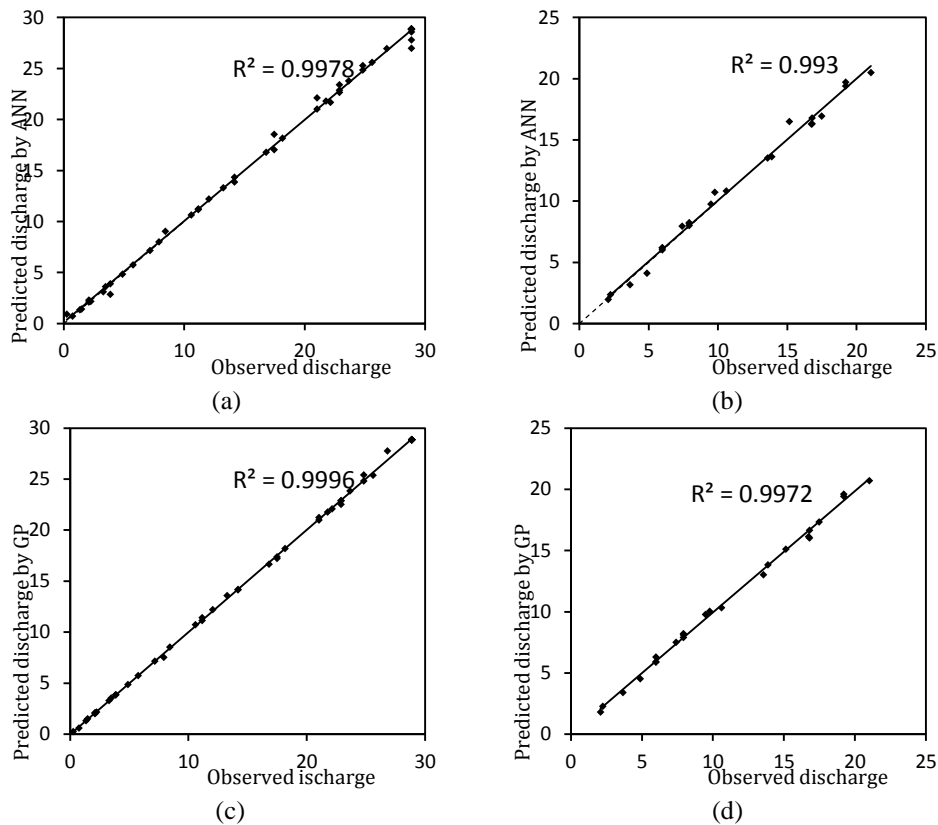
Models	MARE	RMSE	$R^2$
Eq. 5 [8]	0.078	0.922	0.991
Eq. 7 [5]	0.046	0.921	0.992
ANN (this study)	0.075	0.464	0.998
GP (this study)	0.013	0.210	0.9996



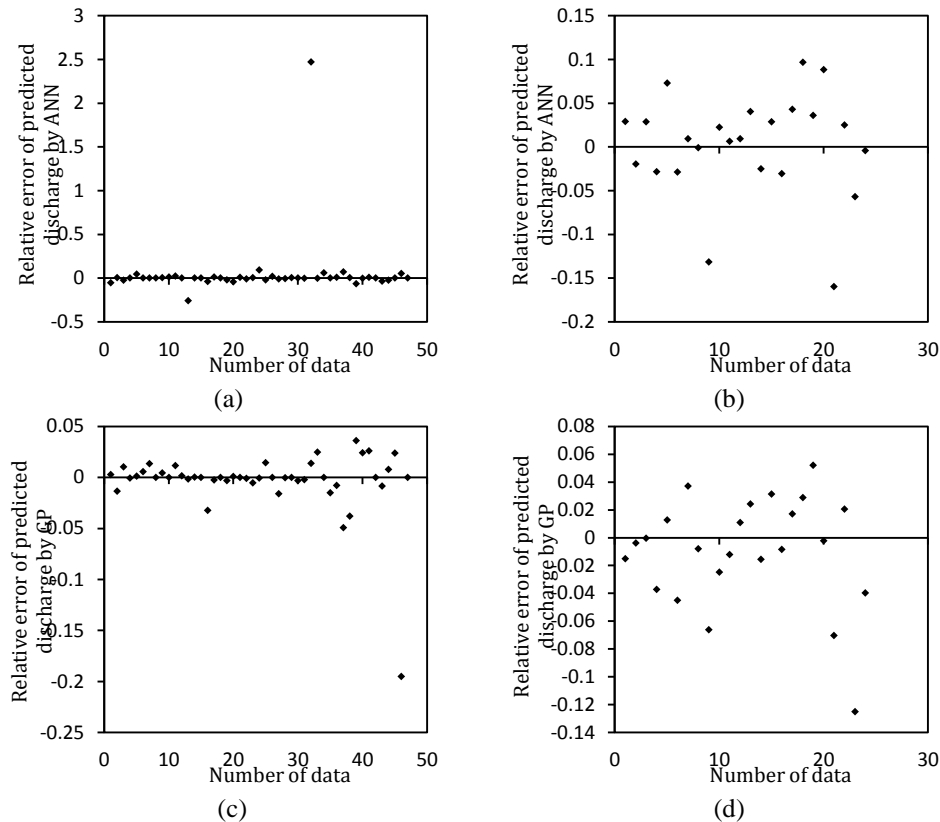
**Table 4: Comparison of the performance of different models for estimating discharge using semicircular weir with semicircular crest for the test data**

Models	MARE	RMSE	R <sup>2</sup>
Eq. 5 [8]	0.079	0.813	0.98
Eq. 7 [5]	0.042	0.619	0.99
ANN (this study)	0.043	0.482	0.993
GP (this study)	0.030	0.311	0.997

The predicted discharges using the AI models were plotted in Figure 5 against the observed ones. As shown, GP predicted closer discharges to the observed ones compared to ANN as it achieved higher R<sup>2</sup> values. Figure 6 depicts  $RE_i$  values for estimating discharge using GP and ANN. As shown, the minimum and maximum values of  $RE_i$  were achieved by GP and ANN for the train data, respectively. Moreover, the range of  $RE_i$  values obtained by ANN is between -0.15 and 0.1 while the corresponding range obtained by GP is between -0.12 and 0.04 for the test data. This demonstrates that GP predicted discharges with a lower error range compared to ANN based on the  $RE_i$  criterion.



**Figure 5: Comparison the performance of AI models for predicting discharge for semicircular weir with semicircular crest: (a) ANN for train data, (b) ANN for test data, (c) GP for train data, and (d) GP for test data.**



**Figure 6: Relative error of predicted discharge for semicircular weir with semicircular crest: (a) ANN for train data, (b) ANN for test data, (c) GP for train data, and (d) GP for test data.**

#### 4. Conclusions

One of hydraulic structures commonly used for flow measurement is weirs. Such measurement using these structures is basically conducted by measuring the height of water over the crest of and relating this height to discharge instead of direct measurement of discharge. Although this technique is streamline in practice, it requires a relation between the water height over the weir crest and the discharge flowing over the weir. This study assesses two AI models named ANN and GP in the calibration of a semicircular weir with two different openings including sharp and semicircular crests. A reliable data set selected from the literature, which was the base of two empirical formulas, was used. The comparison of estimating discharge using AI models and the empirical formulas clearly demonstrate that GP outperforms others in light of the considered criteria.

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