Frequency Dependency of Laboratory Measurement of Maximum Shear Wave Velocity by Bender Elements

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ABSTRACT:
Maximum shear wave velocity is one of the most important dynamic parameters of soils which contributes in estimating the dynamic behaviour of soils through the maximum shear modulus. There are different methods for measuring shear wave velocity either in laboratory or in the field. One of the laboratory methods that recently has become popular due to its simplicity, is use of Bender Elements in soil samples. Contrary to its simplicity, it has several uncertainties in its data interpretation. Frequency at which the shear wave velocity is measured is one of the effective parameters that significantly affects the clearance of received wave. This article presents results from a laboratory investigation into the shear wave velocity measurement of remolded specimens of Firoozkooh sand. Specimens were subjected to 13 different levels of frequency and 11 different levels of confining pressure. Results shows that by increasing the confining pressure, the frequency at which the received wave has the best clearance, increases. It also shows that frequency dependence of soils increases by increasing the confining pressure.

Key words: Shear Wave Velocity, Frequency, Bender Element, Triaxial Apparatus.

1- Introduction
The maximum shear modulus (Gmax) is one of the most important dynamic properties of soil which effectively contributes in evaluating the behavior of soils under dynamic loadings such as earthquake, traffic loads and machine vibrations. There are different methods of measurement either in laboratory or in the field. Laboratory methods could be divided into two groups. In the first group Gmax has been measured based on vibration such as torsional shear and resonant column (TS-RC) tests. The second group of methods has been based on wave propagation like ultrasonic method and Bender Element test. Bender Element method has first proposed by [1]. It rapidly finds its way through laboratory tests due its simplicity in use, respectively low cost, and its non-destructive operation. Bender Elements has been installed and used in different laboratory apparatus such as oedometer [2], [3], torsional shear apparatus [4], resonant column [5], and triaxial apparatus [6]–[9]. Bender Elements are made of two piezoelectric ceramic sheets with central shim of usually ferrous nickel alloys to enhance its strength. Bender Elements operate as electromechanical transducer. When a small voltage is applied to the Bender Element, Bender Element will bend due to the polarization that has induced across its plates and thus the Bender Element will act as a transmitter element. Contrary when a Bender Element bends, a voltage is generated and so it will act as a receiver one [1], [7]. Since the strain induced in soil due to Bender Element movement is in elastic strain range of soils behaviour (less than 10-3%), based on the theory of elastic wave propagation [10], the maximum shear modulus (Gmax) can be derived from equation (1).

\[ G_{\text{max}} = \rho \times V_s^2 \]  

(1)

Where \( \rho \) is soil density and \( V_s \) is shear wave velocity.
By generating a shear wave in the transmitter Bender Element and trace it in the receiver one, it is possible to measure shear wave velocity in soil specimen. Shear wave velocity is derived by following equation:

\[ V_s = \frac{L}{t} \]  

(2)

Where \( L \) = travel length of the shear wave and [7], [11] according to other investigations [7], [11], it is better to assume it as the tip to tip distance between Bender Elements; and \( t \) = travel time of the shear wave and it is equal with time shift between transmitted and received signals. One of the most important obstacle in data interpreting in bender element tests is adequately detecting the time travel. In this study, shear wave velocity of dry Firoozkooh sand, at 13 levels of frequency (3-15 kHz) and 11 different stages of confining stress (10, 20, 30, 40, 50, 100, 150, 200, 300, 400 and 500 kPa) is measured by means of Bender Element equipment which is placed in triaxial apparatus. Maximum shear modulus (Gmax) based on wave propagation theory in elastic media is evaluated as well. The effects of confining stress and frequency are carefully analyzed.

2- Material properties

Firoozkooh no. 161 crushed silica sand in dry condition, is used in this study. It has angular grains with yellow gold colour and uniform grain size distribution. The summary of its principle properties are shown in Table 1. The grain size distribution of this sand and also microscopic photos of its particles are shown in Fig. 1 and Fig. 2.

<table>
<thead>
<tr>
<th>Sand</th>
<th>G_s (MPa)</th>
<th>( e_{max} )</th>
<th>( e_{min} )</th>
<th>D_{50} (mm)</th>
<th>FC (%)</th>
<th>C_u</th>
<th>C_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firoozkooh no. 161</td>
<td>2.65</td>
<td>0.874</td>
<td>0.548</td>
<td>0.27</td>
<td>1%</td>
<td>2.58</td>
<td>0.88</td>
</tr>
</tbody>
</table>

Fig. 1. Grain size distribution curve of Firoozkooh no. 161
3- Test equipment
Tests were performed by a triaxial cell with specimens of 50 mm in diameter and 100 mm in length, which was mobilized with a transmitter Bender Element at pedestal and the receiver one at the top cap. Fig. 3 presents a full view of testing apparatus and Fig. 4 shows a schematic illustration of the triaxial apparatus in conjunction with Bender Element equipment.

Bender Elements have dimensions of 10.5 mm. in width and 18 mm. in length with a protruding depth of 9 mm. There was an integrated data logger which performs as a signal generator, signal receiver, signal amplifier and an oscilloscope, which was capable of producing different kinds of waveforms at a wide range of frequencies and with a maximum voltage of +1 V.
4- Test procedure
The tests were performed in a conventional triaxial apparatus, along with Bender Elements. The transmitter element was mounted in the pedestal and the receiver on the top cap. Cylindrical specimens with dimensions of 50 mm in diameter and 100 mm in height, were prepared using a dry vibration method. Then 4 levels of differential vacuum pressure (10, 20, 30, and 40 kPa) were applied to stabilize the specimen and also to measure the shear wave velocity in low confining stresses. After that, step by step, further 7 levels of isotropic confining stresses (50, 100, 150, 200, 300, 400, and 500 kPa), were applied to each specimen. In each 11 levels of confining stresses (4 levels in vacuum pressure and 7 levels in confining pressure), shear wave velocity was measured by means of transmitting sine waves in 13 different levels of frequencies (3-15 kHz) in the transmitter element and receiving it in the other one.

5- Results and discussion
As it has been mentioned before, one of the most important obstacles in data interpreting in bender element tests is detecting the time travel adequately. There are different methods of interpretation for Bender Element test’s data such as first time of arrival, first peak to peak and cross-correlation method. We used first peak to peak method to obtain time travel of shear wave that has been generated and also received by Bender Elements. In the first peak to peak method, the travel time is obtained by measuring the time distance between the first peak of the transmitted signal and the first peak of the received signal, neglecting the initial distorted part of the curve. Figure 4. Schematically shows peak to peak method of measuring travel time in Bender Element tests.

The results are presented in Figure 5. for a dry specimen of Firoozkooh sand with relative density of 47% which has been applied to 11 levels of confining stress and in each levels of confining stress, it has been tested in 13 different levels of frequency for transmitted sinusoidal signal. The results clearly indicate that maximum shear modulus will increase with increasing the confining stress. It also shows that the frequency dependence will increase with increasing the confining stress of the specimen.

There is an important result that worth’s to be mentioned. As measuring shear wave velocity of the specimens in different confining stresses and frequencies, it has been observed that the clearance of the received signal is not equal in different frequencies for each confining stress. In each confining stress there is limited range of frequencies which will result in best clearance of received signal. We find out that with increasing the confining stress of the specimen, the limited range of the frequencies in which the received signal will have the best
clearance, will increase. The result is logical, as with increasing the confining stress, the specimen will became stiffer and its intrinsic frequency will increase and we know that as much as the frequency of the transmitted signal is near to the intrinsic frequency of the specimen, the clearance of the received signal will be higher. Figure 6. Shows the clearance of received signal in different frequencies for 3 different confining stress.

![Graph showing clearance vs frequency for different confining stresses](image)

Fig. 5. A comparison of maximum shear wave velocity in different levels of confining stress and frequency

![Graph showing frequency of clearance differing of with confining stress](image)

Fig. 6. Frequency of clearance differing of with confining stress

The results are also controlled with empirical equations which had been suggested by different investigators [12]. Maximum shear modulus can be derived from empirical equations which are generally presented in equation no. 3. Different equations are differ from each other by different coefficients. Table 2. Shows these different coefficients for empirical equation.

\[
G_0 = A.F(e) \cdot (\sigma')^n
\] (3)
Figure 7. Illustrates the results of controlling test’s data with different empirical equations. As it shows, the test results have maximum conformity with the equations that is suggested by [13] for 11 kinds of clean sand.

Table 2: Values of coefficients for empirical equation presented by different investigators

<table>
<thead>
<tr>
<th>References</th>
<th>A</th>
<th>H(ε)</th>
<th>n</th>
<th>Soil material</th>
<th>Test method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hardin–Richart (1963)</td>
<td>7000</td>
<td>(2.17 – ε)^2/(1 + ε)</td>
<td>0.5</td>
<td>Round grained Ottawa sand</td>
<td>Resonant column</td>
</tr>
<tr>
<td>Shibata–Socolar (1975)</td>
<td>3300</td>
<td>(2.97 – ε)^2/(1 + ε)</td>
<td>0.5</td>
<td>Angular grained crushed quartz</td>
<td>Resonant column</td>
</tr>
<tr>
<td>Iwasaki et al. (1978)</td>
<td>9000</td>
<td>0.67 – ε/(1 + ε)</td>
<td>0.5</td>
<td>Three kinds of clean sand</td>
<td>Ultrasonic pulse</td>
</tr>
<tr>
<td>Kokusho (1980)</td>
<td>8400</td>
<td>(2.17 – ε)^2/(1 + ε)</td>
<td>0.38</td>
<td>Eleven kinds of clean sand</td>
<td>Resonant column</td>
</tr>
<tr>
<td>Yu–Richart (1984)</td>
<td>7000</td>
<td>(2.17 – ε)^2/(1 + ε)</td>
<td>0.5</td>
<td>Toyoura sand</td>
<td>Cyclic triaxial</td>
</tr>
<tr>
<td>Hardin–Black (1968)</td>
<td>3300</td>
<td>(2.97 – ε)^2/(1 + ε)</td>
<td>0.5</td>
<td>Kaolinite, etc.</td>
<td>Resonant column</td>
</tr>
<tr>
<td>Marcuson–Wahls (1972)</td>
<td>4500</td>
<td>(2.97 – ε)^2/(1 + ε)</td>
<td>0.5</td>
<td>Kaolinite, I_p = 35</td>
<td>Resonant column</td>
</tr>
<tr>
<td>Zen–Umebata (1978)</td>
<td>2000</td>
<td>(4.4 – ε)^2/(1 + ε)</td>
<td>0.5</td>
<td>Bentonite, I_p = 60</td>
<td>Resonant column</td>
</tr>
<tr>
<td>Kokusho et al. (1982)</td>
<td>141</td>
<td>(7.32 – ε)^2/(1 + ε)</td>
<td>0.6</td>
<td>Remolded clay, I_p = 0 ~ 50</td>
<td>Resonant column</td>
</tr>
</tbody>
</table>

*σ: kPa, G0: kPa, **I_p : Plasticity Index

6- Conclusion
In this study, maximum shear modulus of dry samples of Firoozkooh sand in conventional triaxial apparatus in conjunction with Bender Element devices was investigated. Test were performed on 11 different levels of confining stress and 13 different levels of frequency. Time peak to peak method was used to obtain the time travel of transmitted signal of Bender Element. Maximum shear modulus which has been derived from these test was compared with the estimated result from different empirical equations. The outcomes of this study are summarized as follows:
(1) It is obviously derived from the results that with increasing the confining stress, the maximum shear modulus of the specimens will increase too.
(2) The frequency dependence of maximum shear wave velocity will increase with increasing the confining stress.
(3) There is a limited range of frequency for each level of confining stress in which the received signal will be have maximum clearance. As the confining stress increase, this limited range of frequency will be located in higher frequencies. It means that a specimen with higher confining stress will have the clearest received signal in higher frequencies. The results are logical. As we know that with increasing the confining stress of a specimen, it will become stiffer and so its intrinsic frequency will increase too. Furthermore, as much as the frequency of the transmitted signal is near the intrinsic frequency of the specimen, the received signal will have more clearance. (4) As the results are controlled through different empirical equations, it is obvious that the trend of the results are similar and also the results have the most conformity with the coefficients of empirical equation which are suggested by Iwasaki 1978.

7- References