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Buckling Analysis of Frames with Semi-Rigid Connections using Power Series

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ABSTRACT

Analyzing and investigating the elastic behavior of frames after buckling is complicated. When a frame is subjected to a force exceeding the critical load, it begins to undergo large deformations. In this case, the theory of small deformations is no longer valid for the structure and the theory of large deformations should be used. Post-buckling analysis of elastic structures always requires solving a set of nonlinear differential equations based on equilibrium equations. The present work deals with the effect of beam-column joint flexibility on the elastic buckling load of plane steel frames and proposes a simplified approach to the evaluation of the critical buckling load of frames with semi-rigid connections according to the assumptions of the Elastica theory. In this study, the equations are simplified using the power series. The elastic buckling load is found to be strongly affected by semi-rigid joints and reveals that the proposed model is computationally very efficient with the expressions presented being general. The paper makes reference to the other researchers in comparing the results.

Keywords:

Buckling, Frame, Semi-rigid connections, Power series, Elastic theory.

1. Introduction

Analyzing the post-buckling elastic behavior of frames is complex. When a force is applied to a frame more than the critical load large deformations occur. In this case, the theory of small deformations is no longer valid for the structure and we must use the theory of large deformations. Post-buckling analysis of elastic structures always requires solving a set of nonlinear differential equations based on equilibrium equations. In designing members under axial force or axial force and flexural anchor in the structure, in addition to the yielding criterion, the buckling criterion is important. In such a way that if the length of the structural member is long or the member is thin, before yielding, buckling occurs in the member, which requires the member to be checked for possible buckling. Recent Advances in sstructural engineering with new material combinations and advances in mathematical modeling and more precision of numerical calculation tools, has led to the construction of more efficient and slenderer structural members. This is especially important in structures where their effective weight must be minimized, such as space structures, large openings in civil engineering structures, and offshore structures. However, the slenderness of the structural elements makes them more exposed to vibration and buckling, and therefore accurate nonlinear analysis is necessary to ensure the safety of these structures [1]. Post-buckling behavior analysis of beams and plane frames provides important information to design engineers. Equilibrium equations are the most common way to understand the behavior of a structure in post buckling. However, the non-linearity of the structure geometry is a big problem when the structure is under high displacement. Due to this challenge, post-buckling analysis using analytical methods is difficult and numerical methods, especially finite element methods, are used using computer modeling programs such as ANSYS and ABAQUS [2]. So far, many studies have been done on the elastic buckling of plane frames and its methods. The basis of these studies is mainly on the concept of elastic buckling of the frame under neutral equilibrium conditions. It has recently been shown that the initial elastic buckling for a rigid continuous frame can create an unstable state in the structure. Therefore, the performance of the structure after buckling is also very important and should be carefully examined. Built-up members are the most common shear-weak members used in structural engineering nowadays. The calculation of the elastic buckling load of built-up columns for different types of boundary conditions has been carried out by many researchers. The influence of shear deformations has been investigated by Bleich [3], Timoshenko and Gere [4], Aslani and Goel [5], Temple and Elmahdi [6,7], Galambos [8]. Gjelsvik [9] obtained solutions for members with boundary conditions commonly used in the structural industry. Banerjee and Williams [10] explained the reason why the elastic buckling load of members with springs of different rotational stiffness at their ends cannot be derived by the general equation suggested by Engesser [11] and used by Eurocode 3 [12]. Christopher and Bjorhovde [13] conducted analyses of a series of semi-rigid frames, each with the same dimensions, applied loads and member sizes, but with different connection properties, explaining how connection properties affect member forces, frame stability, and inter-story drift. Jaspart and Maquoi [14] described the mode of application of the elastic and plastic design philosophies to braced frames with semi-rigid connections. The buckling collapse of steel reticulated domes with semi-rigid joints was investigated by Kato et al. [15] on the basis of a nonlinear elastic–plastic hinge analysis formulated for three-dimensional beam–columns with elastic, perfectly plastic hinges located at both ends and mid-span for each member. Lau et al. [16] performed an analytical study to investigate the behavior of sub assemblages with a range of semi-rigid connections under different test conditions and loading arrangements. They showed that significant variations in the M–ϕ response had a negligible effect on the load carrying capacity of the column and the behavior of the sub assemblages.

Raftoyiannis [17] presented the effects of the joint flexibility and elastic bracing system on the buckling load. Mageirou and Gantes [18], Gantes and Mageirou [19] proposed a model of an individual column representing a multistory frame where the member contributions converging at the bottom and top ends of the column are represented by equivalent springs. Xu and Liu [20] proposed a method for the stability analysis of semi braced steel frames with the effect of semi-rigid connections and the procedure of evaluating column effective length. Rokhi Shahri et al. [21] investigated the post-buckling behavior of the lateral unbraced frame with the help of Elastica theory. For this purpose, the first step is to analyses a cantilever column by the Maclaurin Series method. By examining the results of the post-buckling behavior of this column with the previous research, the verification of this method has been evaluated. In the following, due to the verification of the method, the large deformations and post-buckling behavior L-shaped frame are investigated. To analyses the frame, it is necessary to solve a nonlinear equation system. The Maclaurin Series method has been used to obtain nonlinear equations. With the help of the equations, the frame deformations diagrams have been plotted. Mathematica software is used to draw charts and solve nonlinear equations. In the following, with the modelling of the frame in Finite Element ABAQUS software, the comparison of the accuracy of the software results with this analysis has been checked and the convergence of the responses has been examined. Bagherzadeh et al. [22] used the power series to simplify the equations. The numerical issues of the critical buckling force are presented for prismatic and non-prismatic columns subjected to end force, and the effectiveness of this approach is verified for buckling analysis of tapered columns and the rate of accuracy is assessed. Many approaches are used to solve limitations to investigate the stability issues of elastic columns with changeable cross-sections subjected to different boundary conditions. The use of the special capability technique, for example, utilizing Bessel functions, emphatically relies upon the type of a customary differential condition with variable coefficients. The present work deals with the effect of beam-column joint flexibility on the elastic buckling load of plane steel frames and proposes a simplified approach to the evaluation of the critical buckling load of frames with semi-rigid connections according to the assumptions of the Elastic theory.

2. Basic assumptions

An L-shaped frame with joint supports at A and C is considered as (Figure 1(a)). The connection of the beam to the column at point B is semi-rigid. Assume spring torsional stiffnessK_θ. We know the larger the K_θ, the tighter the connection and in the limit state $K_\theta \rightarrow \infty$, the connection is rigid. Rigid connection in this case means that the amount of rotation and angle of the end of the column is exactly the same as the amount of rotation and angle of the end of the beam at B. On the other hand, if $K_\theta \rightarrow 0$, the connection tends to the joint which means that there is no relation between the rotation at the end of the column and the end of the beam at B. In this case, the bending moment at the junction is zero. The members of the frame are considered to be prismatic. The column length is L_c and pinned to the support at point A. In case the load is $P \geq P_{\mathrm{cr}}$, where P_{cr} is the critical load of column, the column can be positioned in the equilibrium state as shown in Figure 2. By accepting the simplistic assumption, the axial length change of the column is ignored.

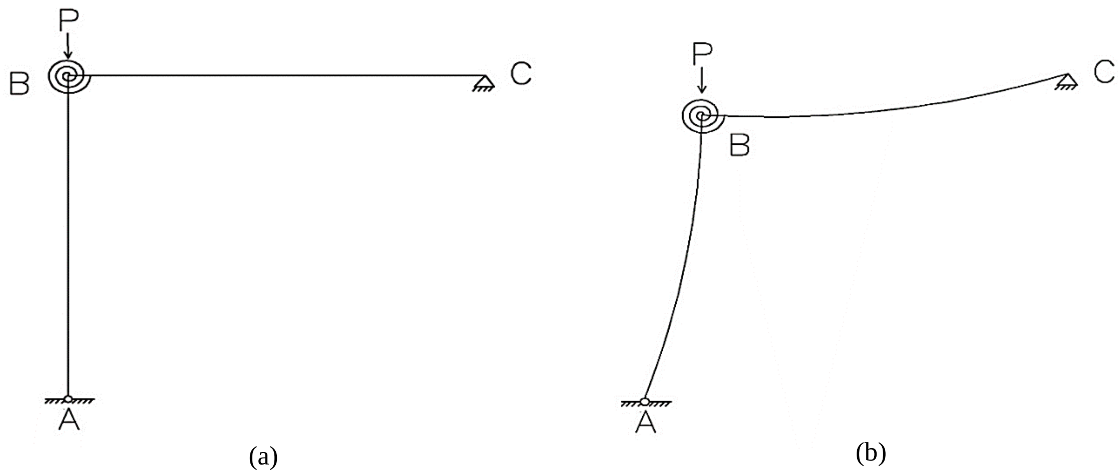

Figure 1. L-shaped frame with a semi-rigid joint.

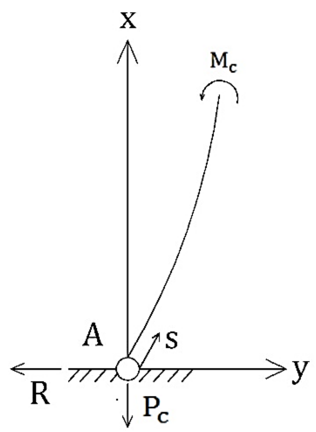

Figure 2. Equilibrium state of column.

The selected coordinate system for the BC beam is as Fig. 3.

Figure 3. Beam coordinate system.

3. Column Analysis

The boundary conditions at point A are:

$$y_c(s=0)=0 \quad (1)$$
$$M_c(s=0)=0 \quad (2)$$

Where y_c is the horizontal displacement and M_c is the internal bending moment. The McLaurin expansion of the slope function, $\theta_c(s)$ is as follows:

$$\theta_c(s)=\sum_{n=0}^{G} a_n^c \frac{s^n}{n!} \quad (3)$$

Where

$$a_n^c=\frac{d^n\theta_c}{ds^n}, \qquad s=0 \quad (4)$$

The zero bending moment in support A gives us the following relation:

$$a_1^c=0 \quad (5)$$

The slope at point A in Figure 2 is θ_{0c} $(a_0^c=\theta_{0c})$ and establishing static equilibrium equations for a part of the structure leads to the following relations.

$$\begin{aligned} M_c &= Rx_c - P_c y_c \\ EI_c\frac{d\theta_c}{ds} &= Rx_c - P_c y_c \end{aligned} \quad (6)$$

Where x_c is vertical displacement, y_c is horizontal displacement and EI_c is the flexural rigidity of the column. To calculate McLaurin expansion coefficients, we derive from the sides of Eq. (6).

$$\frac{d}{ds}\left[EI_c\frac{d\theta_c}{ds}\right]=R\frac{dx_c}{ds}-P_c\frac{dy_c}{ds} \quad (7)$$

Considering that

$$\begin{aligned} \frac{dy_c}{ds} &= Sin\ \theta_c \\ \frac{dx_c}{ds} &= Cos\ \theta_c \end{aligned} \quad (8)$$

Eq. (7) may be written as:

$$\frac{d}{ds}\left[EI_c\frac{d\theta_c}{ds}\right] = RCos\ \theta_c - P_cSin\ \theta_c \tag{9}$$

Where

$$P_c = P + \frac{L_c}{L_b}R \tag{10}$$

Where L_b and L_c are the length of beam and column, respectively. The latter equation gives the following expression with respect to the boundary conditions and according to Eqs. (3) and (5).

$$EI_c a_2^c = RCos\ \theta_{0c} - P_cSin\ \theta_{0c} \tag{11}$$

The calculation of the coefficients a_n^c for $n > 2$ results by sequential derivation from the parties of Eq. (9).

$$a_{n+1}^c = \frac{Rc_n - P_cb_n}{EI_c} \tag{12}$$

Where

$$\begin{cases} b_n = \dfrac{d^{n-1}}{ds^{n-1}}Sin\ \theta_c, \\ c_n = \dfrac{d^{n-1}}{ds^{n-1}}Cos\ \theta_c, \end{cases} \qquad s = 0 \tag{13}$$

The bending moment of the end of the column at point B is obtained from the following equation:

$$M_{cB} = EI_c\frac{d\theta_c}{ds}(s = L_c) \tag{14}$$

Substitution Eq. (3) into Eq. (14) leads to:

$$M_{cB} = EI_c\sum_{n=1}^{G} a_n^c\frac{L_c^{n-1}}{(n-1)!} \tag{15}$$

The slope at the end of the column at point B results from the following equation:

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$$\theta_{cB} = \sum_{n=0}^{G} a_n^c \frac{L_c^n}{n!} \tag{16}$$

4. Beam Analysis

The boundary conditions at point C are:

$$y_b(\xi = 0) = 0 \tag{17}$$
$$M_b(\xi = 0) = 0 \tag{18}$$

Where y_b is the vertical displacement and M_b is the internal bending moment. The McLaurin expansion of the slope function, $\theta_b(\xi)$ is as follows:

$$\theta_b(\xi) = \sum_{n=0}^{G} a_n^b \frac{\xi^n}{n!} \tag{19}$$

Where

$$a_n^b = \frac{d^n\theta_b}{d\xi^n}, \qquad \xi = 0 \tag{20}$$

The zero bending moment in support C gives us the following relation:

$$a_1^b = 0 \tag{21}$$

The slope at point C in Figure 4 is θ_{0b} $(a_0^b = \theta_{0b})$ and establishing static equilibrium equations for a part of the structure leads to the following relations.

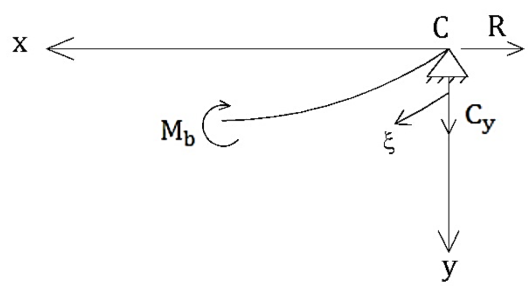

Figure 4. Equilibrium state of beam.

$$M_b = \frac{L_c}{L_b} R x_b - R y_b \tag{22}$$

$$EI_b \frac{d\theta_b}{d\xi} = \frac{L_c}{L_b} R x_b - R y_b \tag{23}$$

Where x_b is horizontal displacement, y_b is vvertical displacement and EI_b is the flexural rigidity of the beam. To calculate McLaurin expansion coefficients, we derive from the sides of Eq. (23).

$$\frac{d}{d\xi}\left[EI_b\frac{d\theta_b}{d\xi}\right]=\frac{L_c}{L_b}R\frac{dx_b}{d\xi}-R\frac{dy_b}{d\xi} \tag{24}$$

Considering that

$$\frac{dy_b}{d\xi}=Sin\ \theta_b$$
$$\frac{dx_b}{d\xi}=Cos\ \theta_b \tag{25}$$

Eq. (24) may be written as:

$$\frac{d}{d\xi}\left[EI_b\frac{d\theta_b}{d\xi}\right]=\frac{L_c}{L_b}RCos\ \theta_b-RSin\ \theta_b \tag{26}$$

The latter equation gives the following expression with respect to the boundary conditions and according to Eqs (19) and (21).

$$EI_b a_2^b=\frac{L_c}{L_b}RCos\ \theta_{0b}-RSin\ \theta_{0b} \tag{27}$$

The calculation of the coefficients a_n^b for $n>2$ results by sequential derivation from the parties of Eq. (26).

$$a_{n+1}^b=\frac{\frac{L_c}{L_b}Rh_n-Rg_n}{EI_b} \tag{28}$$

Where

$$\begin{cases} g_n=\dfrac{d^{n-1}}{d\xi^{n-1}}Sin\ \theta_b, \\ h_n=\dfrac{d^{n-1}}{d\xi^{n-1}}Cos\ \theta_b, \end{cases} \qquad \xi=0 \tag{29}$$

The bending moment of the end of the beam at point B is obtained from the following equation:

$$M_{bB} = EI_b \frac{d\theta_b}{d\xi}(\xi = L_b) \tag{30}$$

Substitution Eq. (19) into Eq. (30) leads to:

$$M_{bB} = EI_b \sum_{n=1}^{G} a_n^b \frac{L_b^{n-1}}{(n-1)!} \tag{31}$$

The slope at the end of the beam at point B results from the following equation:

$$\theta_{bB} = \sum_{n=0}^{G} a_n^b \frac{L_b^n}{n!} \tag{32}$$

5. Equilibrium of joint B

In torsion springs one can write:

$$M_\theta = K_\theta \Delta\theta \tag{33}$$

Where $K_\theta$the stiffness of the torsion is spring and $\Delta\theta$ is the amount of rotation. The above equation can be written as follows for node B:

$$M_{cB} - M_{bB} = K_\theta(\theta_{cB} + \theta_{bB}) \tag{34}$$

According to Figure 1(b), the following relation is obtained by writing the bending equation about point B:

$$P_c y_{Lc} + Rx_{Lc} + Ry_{Lb} - \frac{L_c}{L_b} Rx_{Lb} = 0 \tag{35}$$

Where y_{Lc} and x_{Lc} are horizontal and vertical displacement of the column, respectively and y_{Lb} and x_{Lb} are vertical and horizontal displacement of the beam, respectively at point B which can be obtained with the following equations using the McLaurin series.

$$y_{Lc} = \int_0^{L_c} \text{Sin}\ \theta_c\ \text{ds} = \int_0^{L_c} (\theta_c - \frac{{\theta_c}^3}{3!} + \cdots)\ \text{ds} \tag{36}$$

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$$x_{Lc} = \int_0^{L_c} \text{Cos}\ \theta_c\ \text{ds} = \int_0^{L_c} (1 - \frac{{\theta_c}^2}{2!} + \cdots)\ \text{ds}$$

$$y_{Lb} = \int_0^{L_b} \text{Sin}\ \theta_b\ \text{d}\xi = \int_0^{L_b} (\theta_b - \frac{{\theta_b}^3}{3!} + \cdots)\ \text{d}\xi$$

$$x_{Lb} = \int_0^{L_b} \text{Cos}\ \theta_b\ \text{d}\xi = \int_0^{L_b} (1 - \frac{{\theta_b}^2}{2!} + \cdots)\ \text{d}\xi$$

The substitution of the coefficients a_n^c and a_n^b in terms of θ_{0c}, θ_{0b}, P and R in the Eqs. (34, 35) provides the following equations.

$$\begin{cases} f(P, R, \theta_{0c}, \theta_{0b}) = 0 \\ g(P, R, \theta_{0c}, \theta_{0b}) = 0 \end{cases} \tag{37}$$

According to the concept of neutral equilibrium in structural stability in the limit state$(\theta_{0c}, \theta_{0b}) \rightarrow 0$, the applied load P tends to P_{cr}.

6. Numerical example

This example previously published [18, 19] is presented, for which the proposed approach is demonstrated and the results are compared and validated. Consider the frame of Figure 5 with a single span $L = 20$ mand height $h = 10$ m, having a column with HEB360 cross-section and a beam with IPE400 cross-section. The characteristics of the structural elements are given below

For the beam $\begin{Bmatrix} EI=48573\ kN.m^2 \\ EA=896490\ kN \end{Bmatrix}$ and for the column $\begin{Bmatrix} EI=90699\ kN.m^2 \\ EA=1272600\ kN \end{Bmatrix}$

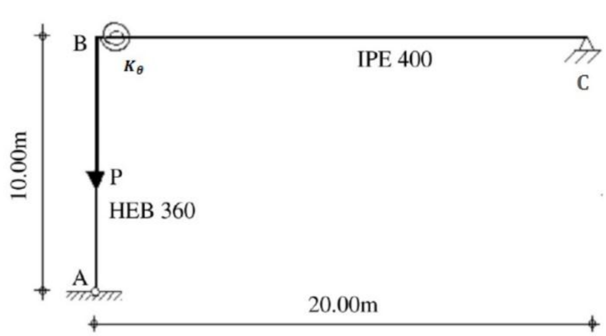

Figure 5. The frame of example.

Table 1. Comparison of the critical load values for G=12.

Methods	P_{cr}(kN)	$\frac{P_{cr} - P_{cr}, MEF}{P_{cr}, MEF}$(%)

F.E.M- MSC-NASTRAN [18]	9027.06	0
Ref. [18]	9027.30	0.0026
Present study	9034.24	0.0795

The column is considered to be pinned at the base. A concentrated load P is imposed on the beam–column joint. The beam–column joint is considered to be semi-rigid, with a rotational stiffness of $K_\theta = 150\ \mathrm{kN.m/rad}$. The frame is made of steel with Young's modulus $E = 210\,000\,000\ kN/m^2$. The frame of Figure 5 is analyzed using the proposed formulation by Wolfrom Mathematica software [23] compared to those given by the Ref. [18]. Table 1 gives the buckling load values obtained for different methods.

7. Conclusion

The presented example and comparison between the results gives a good correlation, suggesting that the proposed model is adequate and may be a useful tool in the analysis of steel frames with semi-rigid joints. This method can be introduced as a suitable method for analyzing post-buckling behavior of steel frames. Although achieving high accuracy requires the use of a large number of sentences in the McLaurin series, software limitations in some cases prevent this goal from being achieved, however the present method is much simpler to use.

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