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Analysis of Clamped Circular Plates with Large Deflections under Uniform Loading using Point Collocation Method

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ABSTRACT

When the plate is under large lateral loads the maximum deflection of the thin plate is equal or larger than the thickness of plate. Because of these large displacements the mid-plane stretches, and hence the in-plane tensile stresses developed within the plate stiffen and add considerable load resistance to it. Due to the restrictions of analysis methods, researchers suggest using numerical methods for these kind of problems. Numerical methods includes Finite element method, Boundary element method, Finite difference method, Point collocation method, Ritz's method, Galerkin's method, etc. Some of numerical methods trying to change the problem from solve partial differential equation to solve a system of differential equations. In this paper circular plate with clamped edges under uniform loading and large deflections is researched by using point collocation method. So large deflection of plate is assumed as a function of small deflection of plate. This assumption convert the problem from solve partial differential equation to solve a system of differential equations that is easy to solve and has good convergence rate. Finally the results of this method are compared with the results from analyzing model in ABAQUS software and Timoshenko's exact solution.

Keywords:

Circular plate, Large deflection, Point collocation method, Finite element modeling, Timoshenko's exact solution.

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1. Introduction

Structures that bear lateral loads, such as end plates and caps of pressure vessels, pump diaphragms, clutches and turbine discs, usually have a circular shape in practice. Therefore, many important applications of plate theory are included in the range of derived formulas for circular plates. If the deflection of the plate is small compared to its thickness, an approximate but acceptable theory for the bending of the plate under external loading can be provided using the following hypotheses:

1- There is no deformation in the middle plane of the plate, this plane remains neutral when bending.

2- The points of the plate that are perpendicular to the middle plane before bending remain in the same place after bending the plate.

3- The stresses perpendicular to the middle plane of the plate can be ignored.

By using these hypotheses, all stress components can be shown in terms of plate stiffness, which itself is a function of the x and y coordinates of the plate. The second equivalent assumption is that the effect of shear forces on the rise of the plates is not taken into account. This assumption is usually acceptable, but in some cases (such as the case where there is a hole in the plate), the effect of shear force is important, and in this case, it is necessary to make corrections in the theory of thin plates. Now, if in addition to lateral forces, external forces also act on the middle plane of the plate, the first assumption will not be true and it is necessary to consider the effect of the stresses acting on the middle plane of the plate on its bending. Adding some terms to the differential equation makes this problem possible. Only if the plate bending is reversible, the first assumption is completely acceptable. In other cases, plate bending is associated with mid-plate strain. But the calculations show that if the rise in the plate is insignificant compared to its thickness, the corresponding stresses in the plate can be ignored. In this case, nonlinear equations will be obtained and solving the problem will be much more difficult. If there is a large rise, a difference should be made between the non-moving edges and the edges that move freely in the plane of the plate and have a significant effect on the magnitude of the rise and the stresses of the plate. Complementary stresses, which are larger in number than the others, act against the lateral loads due to the curvature created in the middle plane of the plate. Therefore, part of the incoming load is transferred by hardness and part of it by membrane action of the plate. As a result, very thin plates that have little resistance to bending act as membranes; Except for the region of the edges of the network, which can be bent due to the existence of boundary conditions in the plate. In the case of reversibility of plate bending, an exception should be made for surfaces such as cylindrical surfaces. Because the rise in such a plate may be proportional to its thickness, without necessarily creating membrane stresses and contradicting the linear nature of bending theory. The creation of membrane stresses in a plane is provided that the domain of the plane is immobile in the direction of the plane and its slope is large enough. Therefore, in plates with low rise, the membrane forces created by the non-moving edges of the plate can be ignored [1]. In this research, circular plate with clamped edges under lateral loads and large deflections is researched by using point collocation method. So large deflection of plate is assumed as a function of small deflection of plate. This assumption convert the problem from solve partial differential equation to solve a system of differential equations that is easy to solve and has good convergence rate. Finally the results of this method are compared with the results from analyzing model in ABAQUS software [2] and Timoshenko's exact solution.

2. Background of research

The nonlinear vibrations of plates were investigated by Chia in 1980 [3]. After Kirchhoff [4] founded the classical linear plane theory, Von Karman [5] expanded Kirchhoff's theory. The study of nonlinear plate dynamics was first investigated by Chu and Herrmann [6], in this study, the vibration of rectangular plates with a simple support was investigated. Mindlin plate theory [7] calculates shear strains, which is suitable for composite and thick plates. Leung and Mao [8] investigated rectangular plates with simple support with moving edges and also with non-moving edges using Galerkin's method. Kadiri and Beammar [9] introduced a simple analytical model for checking plates by using Chu and Herman's method. Berger [10] simplified the theory of non-linear plane by omitting the terms related to strain energy. Prathab and Pandalai [11] obtained favorable results for the nonlinear theory of plates by combining rotational inertia and correcting it for shear. Yosibash and Kirby [12] investigated three different types of conditions of the nonlinear theory of plates: in the first type, the sentences related to rotational inertia were omitted, in the second type, the rotational inertia was omitted and also the sentences depending on the time, the desired model was simplified. In the last type of condition, it was analyzed using all the deleted sentences. Amabili [13] compared the experimental and analytical results by applying different boundary conditions. He [14] Ritz, Leung and Mao energy method [7] Galerkin method, Ribrio [15] finite element model and Yuan et al. [16] applied the approximate method to analyze the nonlinear system governing the plates. Fourier series was used by Levy [17] to analyze simply supported plates under different boundary conditions. Timoshenko's book Theory of Plates and Shells [18] is considered as the first reference for the problems of plane orientation in most research works. Dastjerdi and Yazdanparast [19] applied SAPM method and the nonlinear partial differential equations have been transformed to the nonlinear algebraic equations system. Then, the nonlinear algebraic equations have been solved by using Newton–Raphson method. The obtained results of this study have been compared with the results of other references and the accuracy of the results has been shown. The effect of some important parameters on the results such as the location of the circular hole, the ratio of major to minor radiuses of elliptical plate, the size of circular hole and boundary conditions have been studied. It is concluded that applying the presented method is very convenient and efficient. So, it can be used for analyzing the mechanical behavior of elliptical plates, instead of relatively complicated formulations in elliptic coordinates system. Rokhi Shahri et al. [20] investigated the post-buckling behavior of the lateral unbraced frame with the help of Elastica theory. For this purpose, the first step is to analyses a cantilever column by the Maclaurin Series method. By examining the results of the post-buckling behavior of this column with the previous research, the verification of this method has been evaluated. In the following, due to the verification of the method, the large deformations and post-buckling behavior L-shaped frame are investigated. To analyses the frame, it is necessary to solve a nonlinear equation system. The Maclaurin Series method has been used to obtain nonlinear equations. With the help of the equations, the frame deformations diagrams have been plotted. Mathematica software is used to draw charts and solve nonlinear equations. In the following, with the modelling of the frame in Finite Element *ABAQUS* software, the comparison of the accuracy of the software results with this analysis has been checked and the convergence of the responses has been examined. Bagherzadeh et al. [21] used the power series to simplify the equations. The numerical issues of the critical buckling force are presented for prismatic and non-prismatic columns subjected to end force, and the effectiveness of this approach is verified for buckling analysis of tapered columns and the rate of accuracy is assessed. The elastic buckling force of elastic structures shows that the introduced model is computationally extremely efficient with the details presented in general. This paper should be a basic reference to compare the results with other researches.

3. Modelling process

3.1. Finite element modeling

In this section, the simulation of the plate will be done by using *ABAQUS* software. The circular plate is modelled under uniform loading. In the following, in order to compare the responses obtained from the method investigated in this paper, due to the lack of Timoshenko results in loadings greater than 11 ($qa^4/Eh^4 > 11$), the software simulation is utilized in order to create a criterion for measuring the correctness of the responses. In the following, all the steps required for this task are given along with the Figures 1 to 5.

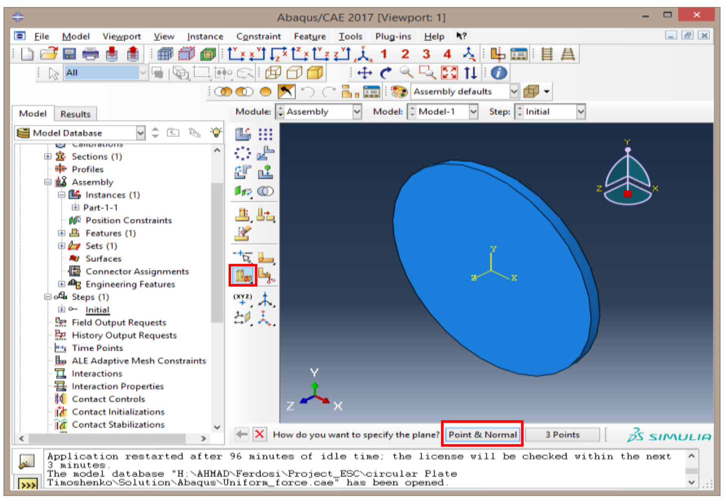


Figure 1. The modeled of circular plate.

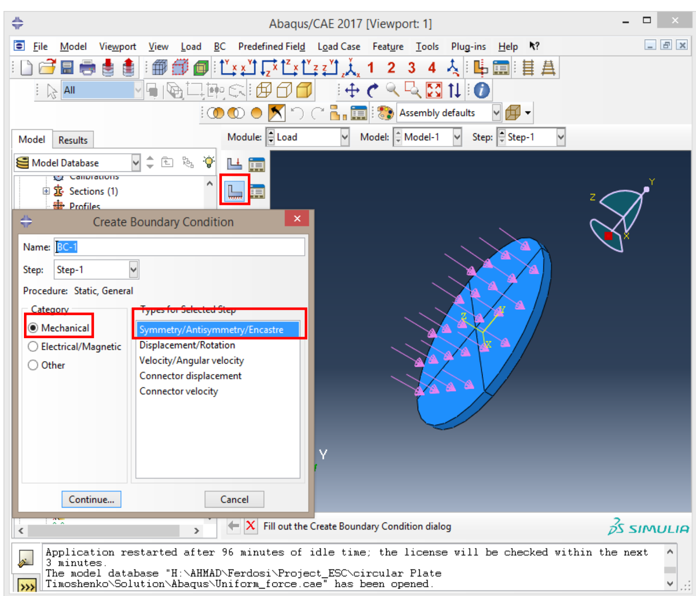


Figure 2. The definition of boundary conditions.

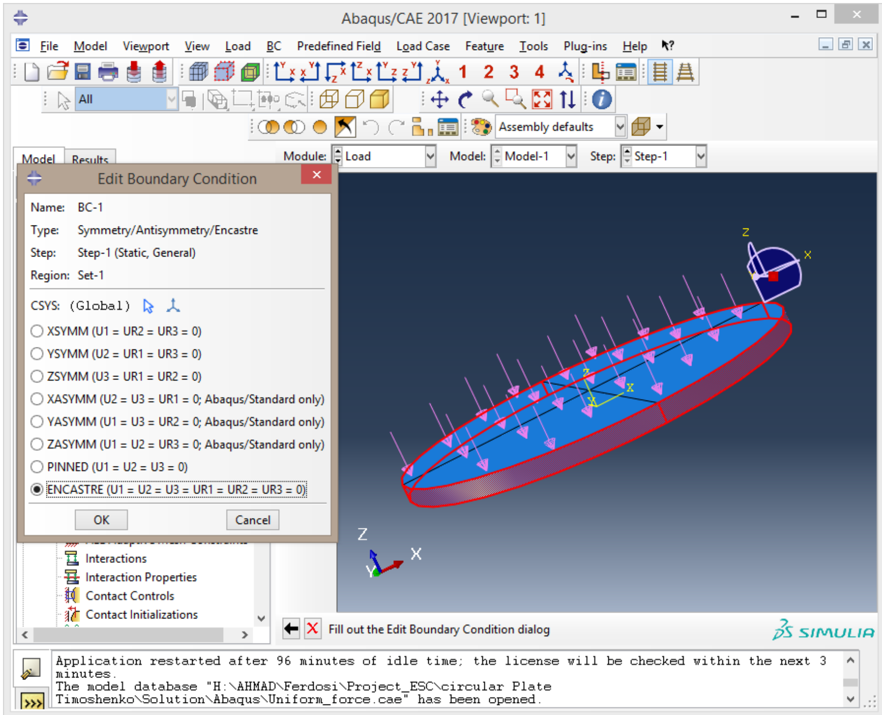


Figure 3. The kind of boundary conditions.

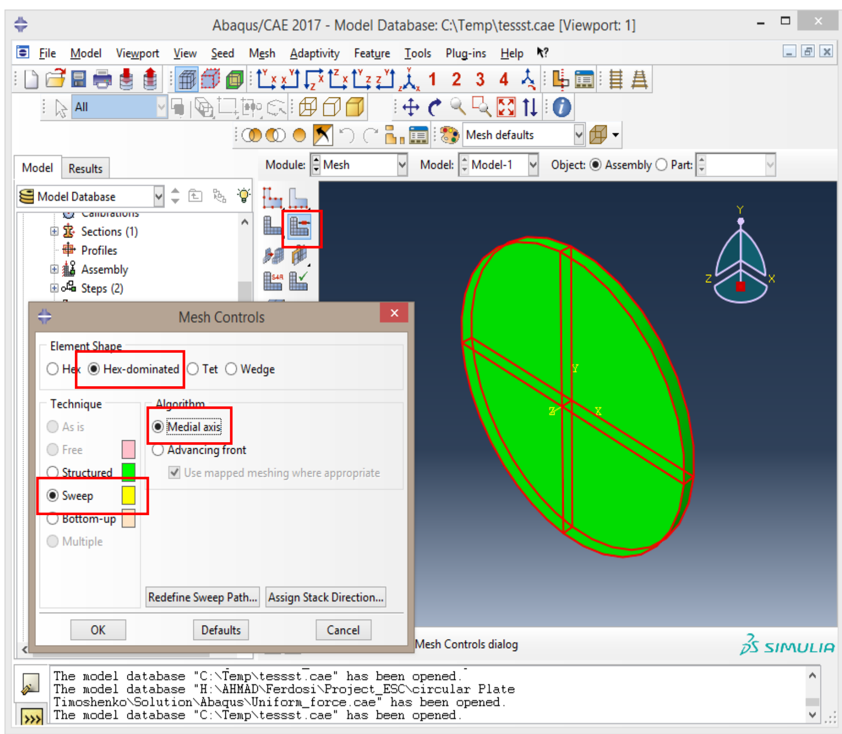


Figure 4. The mesh settings of modeled circular plate

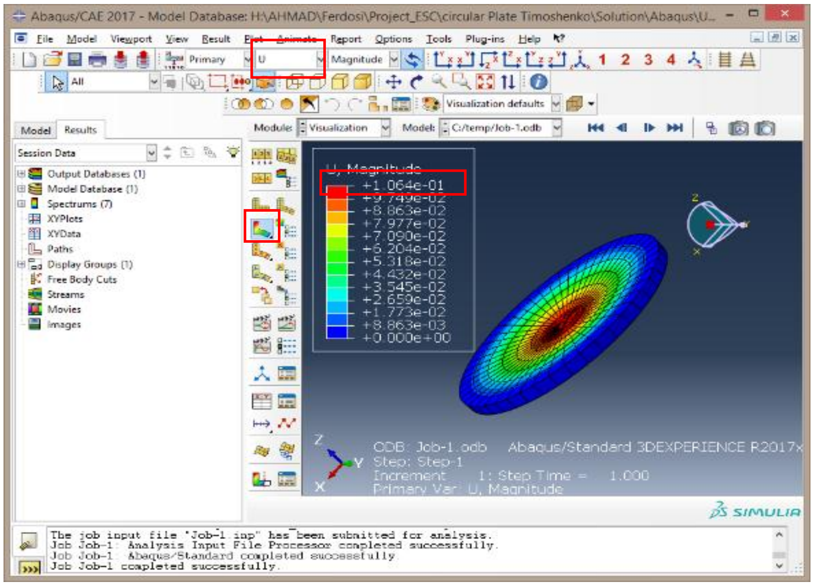


Figure 5. The counter of displacement of modeled circular plate.

3.2. Point collocation method

In this section, the circular plate with retaining edges under loading and large deformations has been investigated using the approximate method of point collocation. Point collocation is a method that is used to construct a system of algebraic equations without using a predefined grid to discretize the domain. At first, the domain and boundary of the problem are displayed arbitrarily by a set of points, in order to obtain the variables of the solution field, a function of a suitable shape with unknown coefficients is assumed and placed in the governing equations of the problem, this assumption leads to solving differential equations with partial derivatives It will convert the machine to solve algebraic equations. Despite its simple analytical foundations, this method is rarely used today because the results of the analysis depend heavily on the selection of domain points. In this research, it was tried to control and legalize the way of choosing points by formulating the points. The problem in question is a circular plate which is investigated under two types of loading. At first, the plate was considered with a uniform loading q, and in order to approximate the shape function, the deflection of the plate in the case of large deformations was considered using the McLaren series, a coefficient of the deflection of the plate in the case of small deformations. Next, the effect of various factors in the point collocation method, such as the number of points, their location, and how they are placed in relation to each other, was investigated, and at the end, the same process was repeated for the plate with point loading in its center. In the following, the differential equations governing circular plates with large deformations are equal to Equation (1):

$$
\begin{aligned}
&\frac{d^2u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{u}{r^2}=-\frac{1-\nu}{2r}\left(\frac{dw}{dr}\right)^2-\frac{dw}{dr}\frac{d^2w}{dr^2}\\
&\frac{d^3w}{dr^3}+\frac{1}{r}\frac{d^2w}{dr^2}-\frac{1}{r^2}\frac{dw}{dr}=\frac{12}{h^2}\frac{dw}{dr}\left[\frac{du}{dr}+\nu\frac{u}{r}+\frac{1}{2}\left(\frac{dw}{dr}\right)^2\right]+\frac{1}{Dr}\int_0^r qrdr
\end{aligned}
\qquad (1)
$$

Where u is the lateral deformation, w is the transverse deformation and D is the bending stiffness of the plate. Due to the symmetry of the plane with respect to any hypothetical axis, the coordinates w and u are independent of the angle (Teta). The bending stiffness equation is expressed as follows:

$$D=\frac{Eh^3}{12\left(1-\nu^2\right)} \tag{2}$$

Due to the existence of central symmetry and the dependence of the yield rate of the plate on r, therefore the yield ratio resulting from large to small deformations will be an unknown function in terms of r, so the function was assumed as a power series in terms of r and then the function of the appropriate shape was obtained such as Equation (3):

$$\frac{w_L}{w_s}=\sum_{n=0}^{N} a_n r^n \tag{3}$$

In order to solve the resulting system of nonlinear equations using the point collocation method, the solution of the problem as a function is considered as follows:

$$W_L=w_s\sum_{n=0}^{N} a_n r^n \tag{4}$$

Which W_l indicate the large vertical displacement of the plate, W_s small displacement of the plate and a_n unknown coefficients that can be calculated using the point collocation method. The small displacement of the plate is equal to Equation (5):

$$w_s=\frac{q}{64D}\left(a^2-r^2\right)^2 \tag{5}$$

The boundary conditions for the clamped plate are as follows:

$$\begin{aligned} r=0 \quad &\Rightarrow \frac{dw}{dr}=0 \ , \ \sigma_r \ \text{is limited} \\ \text{r=a} \quad &\Rightarrow w=u=0 \ , \ \frac{dw}{dr}=0 \ , \ \sigma_r=\sigma_0 \end{aligned} \tag{6}$$

That the membrane stresses (σ_0) created at the edge of the receiver are assumed to be uniform.

4. Results and discussion

In the first step, in order to check the correctness of the point collocation method, the convergence of the solution is checked for two different times. As seen in Figure 6, with the increase in the number of terms of the power series, the accuracy of the calculations increases and gradually the solution of the equation converges towards a constant number, which is a proof of the correctness of the solution of the equation. In Figure 6 (a), the changes are drawn for dimensionless load value 3 and in Figure 6 (b), the diagram is drawn for dimensionless load 10.

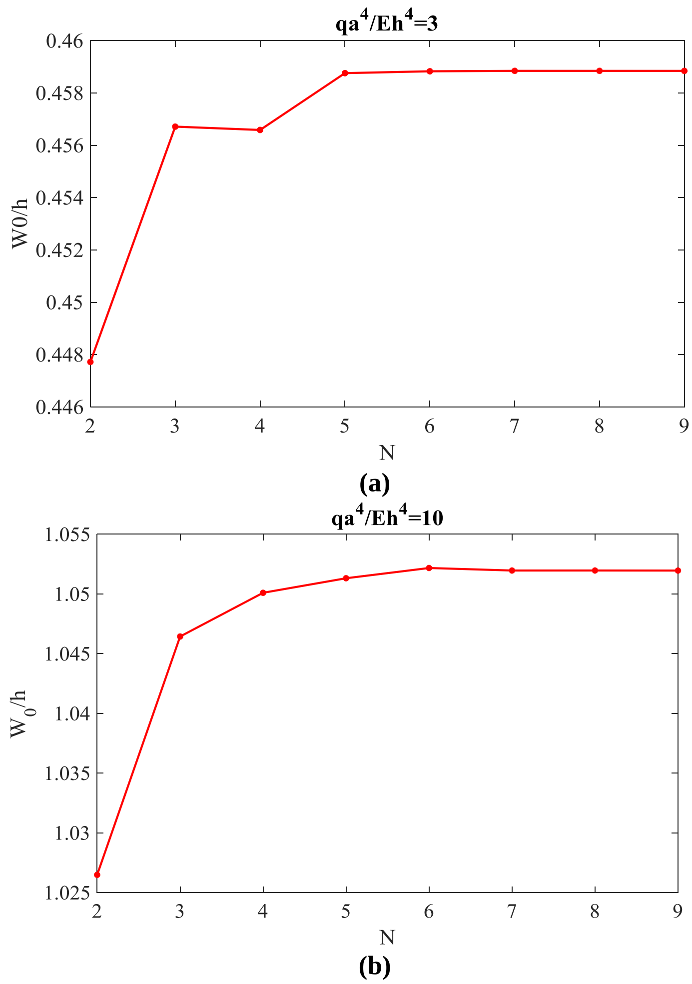


Figure 6. Changes in the transformation of large forms according to the number of power series sentences.

In this part, the obtained responses of the proposed method and Timoshenko's large deformations are compared with each other. At first, the solution method in Timoshenko's large deformations book is written as a computer program and its results are shown in Figure 7.

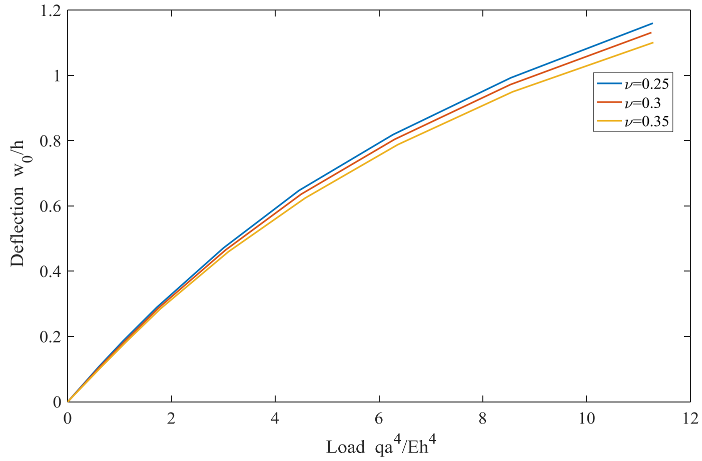


Figure 7. Changes of deformation with load in Timoshenko's method.

Now, these values with the results of the proposed method are compared with each other. In Figure 8, the slight difference between these two methods is clear with $\nu = 0.3$. (N is the number of sentences involved in the power series). In the following, in order to investigate accurately, the results are presented in Table 1.

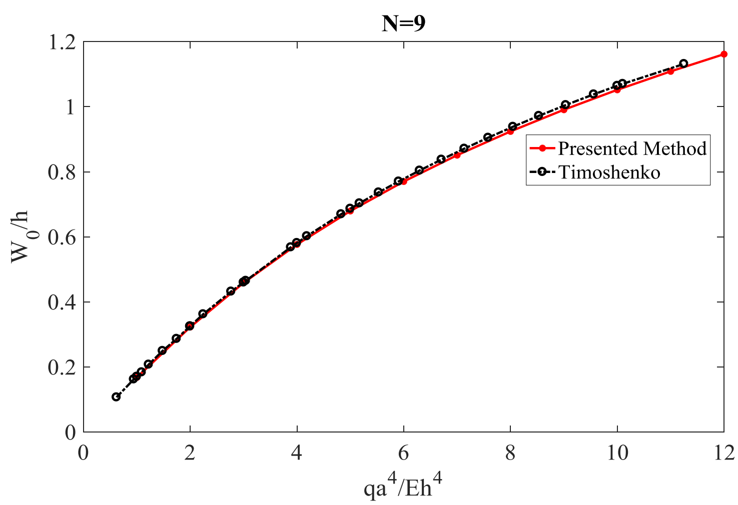


Figure 8. Comparison of Timoshenko's results and the present work

Table 1. Comparison of Timoshenko's results and the present work.

qa^4/Eh^4	Present Work w_0/h	Timoshenko w_0/h	Error (%)
1	0/168	0/1706	**1/52**
2	0/3228	0/3258	**0/92**
3	0/4588	0/4603	**0/32**
4	0/5768	0/5815	**0/80**
5	0/6796	0/6868	**1/04**
10	1/0520	1/0644	**1/16**

Considering the reasonableness of the responses and the correctness of the chosen method, in this part the conditions of using fewer points and how they are placed relative to each other are applied.

4.1. To calculate exact response using two points

Due to the convergence of the responses in the number of points higher than 7, in order to obtain the exact response using the two points, the location of the points becomes important in relation to each other. To find these points, due to the strong dependence of the responses on the load value, the dimensionless load is started by value 1 ($qa^4/Eh^4=1$) and continuing until it is possible to find the response. one of the points as the center and the second point, as you can see in Figure 9, is considered as a ratio of the radius of the plate. As it can be seen, for this amount of load, there is no encounter with the response obtained by Timoshenko's method. This process is repeated for larger loads, too. A similar diagram for dimensionless load value 5 is given in Figure 10 ($qa^4/Eh^4=5$). As it can be seen, considering the second point to the coordinates of 0.65 of the radius of the plate, the exact response of Timoshenko can be obtained.

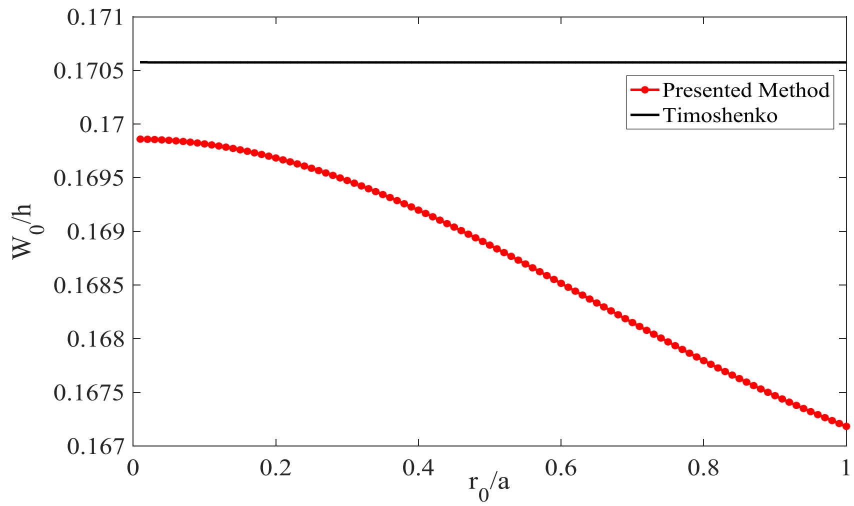


Figure 9. Dimensionless displacement changes according to the second point coordinate in dimensionless load value 1 ($qa^4/Eh^4=1$).

Advance Researches in Civil Engineering
ISSN: 2645-7229, Vol.5, No.1, pages: 70-85

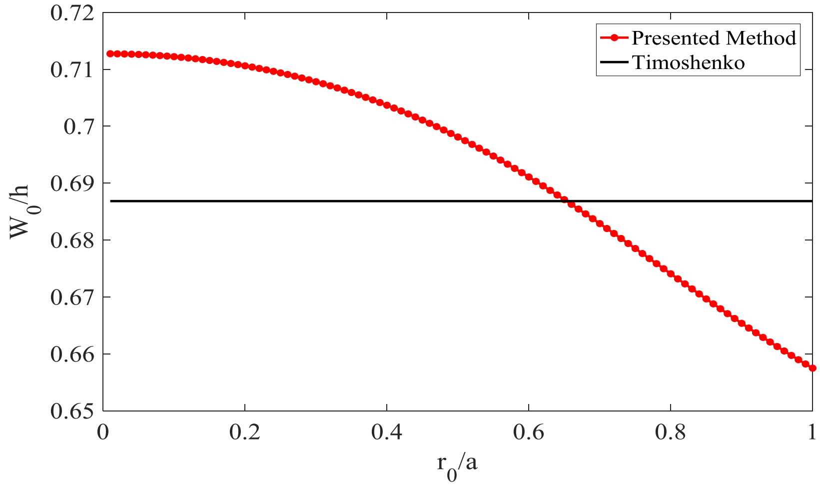


Figure 10. Dimensionless displacement changes according to the second point coordinate in dimensionless load value 5 ($qa^4 / Eh^4 = 1$)

The mentioned method for loadings between 1 and 11 is considered and by checking the collision point with the exact response, a graph according to the loading and the characteristic of the second point is obtained. This diagram is drawn in Figure 11. It can be seen from the diagram that the two-point power series can only be used for loads smaller than 11. Due to the specific trend of these points in the range of 3 to 11 values, it was fitted and in the following, the obtained Equation (7) can be used to calculate the desired point.

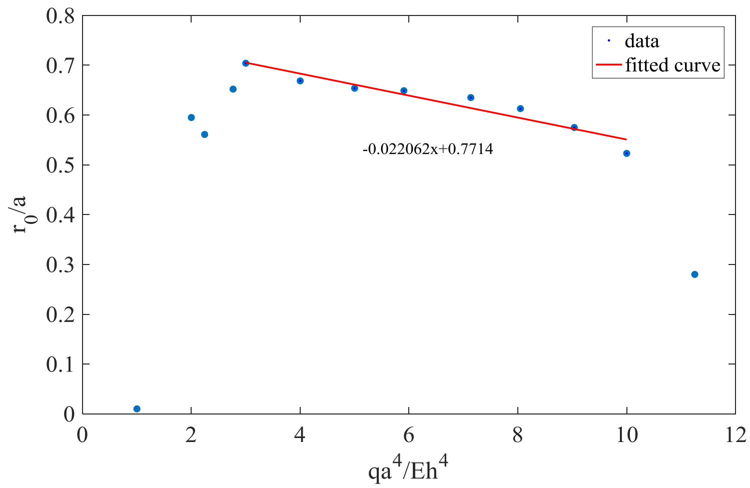


Figure 11. Changes in the characteristic of the second point according to the amount of loading.

For $qa^4 / Eh^4 < 11 \quad \Rightarrow \quad r_0 / a = -0.022x + 0.77$ (7)

4.2. To calculate the exact response using three points

In the previous part, a relationship was obtained for loads less than 11. Now, to calculate the solution of the equation for loads greater than 11, the number of points involved in the power series is increased by one. Due to a more detailed and logical investigation, the convergence solution as the comparison criterion is considered and compare this solution with the solution obtained from the modeled system in *ABAQUS*. In Figure 11, you can see changes in the dimensionless displacement of the plate according to the third point, in 10 different points. The selection of points is such that the first point of the center of the plate, the second numerical point of the points given in the figure guide and the third point of collision with the convergence solution can be selected.

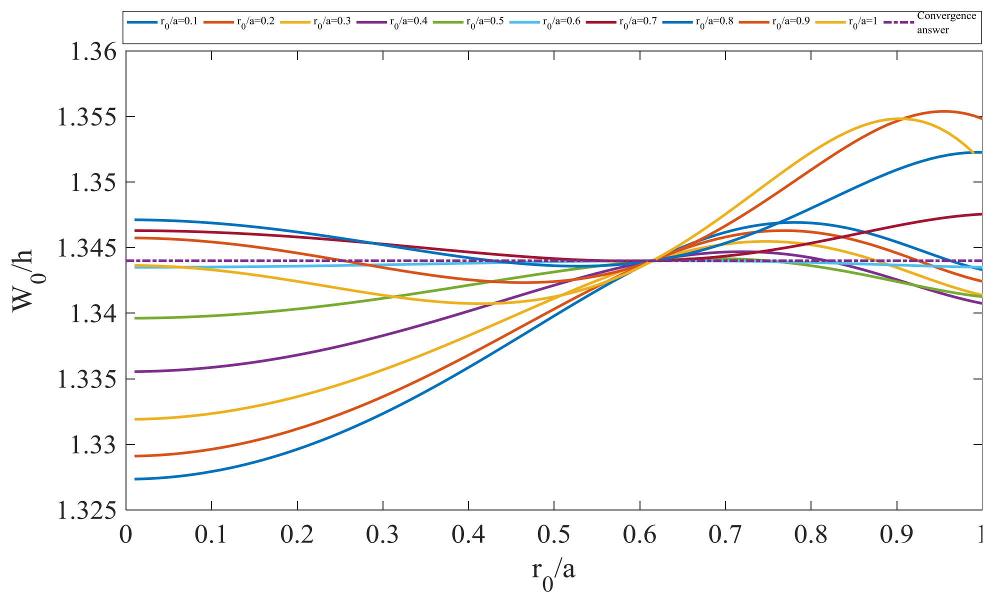
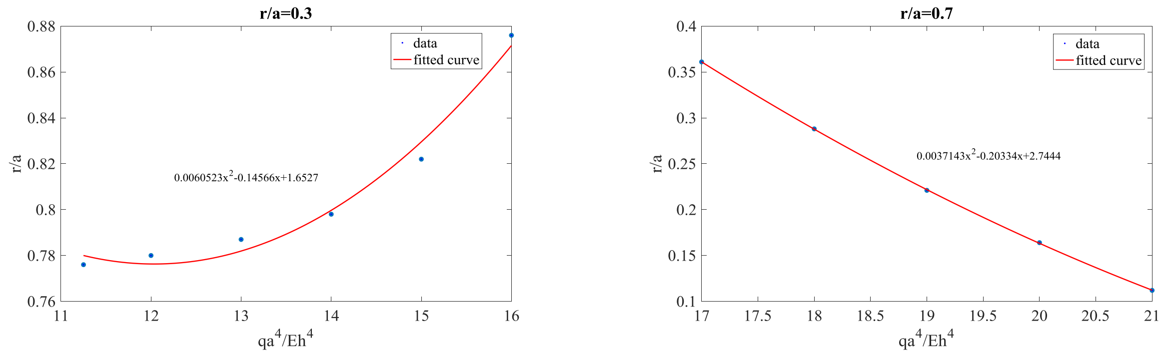


Figure 12. Dimensionless displacement changes according to the third point coordinate in dimensionless load value 16 ($qa^4 / Eh^4 = 16$).

Through the investigations, we found that the choice of the second coordinate depends on the load value; thus, for loads between 11 and 16, from 0.2 to 0.5 points, and for loads between 17 and 21, from 0.7 to 0.9 points, it gives the best response. Figure 13 shows examples of the graphs obtained for the second coordinate of 0.3 and 0.7 (respectively (a) and (b)) according to loading. In order to simplify the use of these diagrams, we have adapted a diagram to discrete points. The full description of these relationships for different points is given in Table 2.



(a) (b)

Figure 13. Specific changes of the third point according to loading.

Table 2. The third ordinate relationships according to loading.

qa^4/Eh^4	**The second point**	**The third equation**
11-16	0.2	0.0094x^2-0.22x+2.12
	0.3	0.006x^2-0.15x+1.65
	0.4	0.0012x^2-0.03x+0.96
	0.5	-0.0072x^2+0.17x-0.21
17-21	0.7	0.0037x^2-0.2x+2.7
	0.8	0.0002x^2-0.011x+0.54
	0.9	-0.0071x^2+0.29x-2.5

$$x = {qa^4}/{Eh^4}$$

Table 3. The error of convergence response from finite element solution

qa^4/Eh^4	**Present Work** w_0/h	**Timoshenko** w_0/h	**Error (%)**
11.25	1.1223	1.136	**1.206**
12	1.1616	1.176	**1.224**
13	1.2112	1.227	**1.288**
14	1.2579	1.275	**1.341**
15	1.3021	1.321	**1.431**
16	1.344	1.364	**1.466**
17	1.384	1.405	**1.495**
18	1.4221	1.445	**1.585**
19	1.4587	1.483	**1.639**
20	1.4938	1.519	**1.659**
21	1.5275	1.5538	**1.693**

The use of the obtained relations leads to the solution of convergence using three points. The error of solving the equations with three points compared to the numerical solution of *ABAQUS* is compiled in Table 3. For loads greater than 21, it is not possible to use three points. To solve the equation for these values is possible by using more points.

Advance Researches in Civil Engineering
ISSN: 2645-7229, Vol.5, No.1, pages: 70-85

4.3. To calculate the exact response with convergence points

For loads greater than 21, we have to use more points. In this section, we will examine the appropriate and optimal range for selecting points according to the amount of loading. In order to find the optimal range of point selection, we start with a load value greater than 21 and consider the starting point coefficient to be 0.1 (in all the relations ahead, the zero point and the radius are applied by default, that is, to solve 9 points to only 7 points is no longer needed). In the following, we consider b as the end point and divide this interval into six equal parts (we considered the number of necessary points as 9 points, equivalent to convergence points). By changing the value of b, we obtain the response of the machine of equations and compare it with the response obtained from convergence with many points. It can be see that all the mentioned steps in Figure 14. It is clear that the best solution occurs at the point 0.6, which means that we divide the interval from 0.1 to 0.6 into 6 equal parts and together with the zero and one points, we solve the system of equations and this solution is the closest solution. is the amount of convergence.

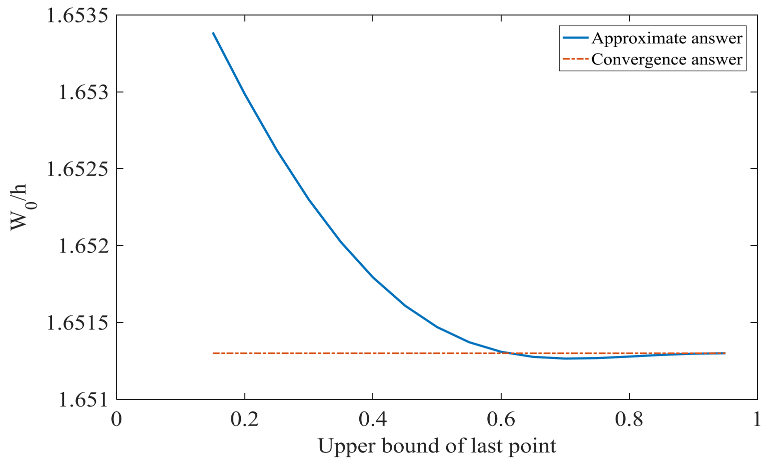


Figure 14. The last point's characteristic according to the dimensionless displacement of the plate in the load value 25 ($qa^4 / Eh^4 = 25$).

We repeat the same process for larger loads and plot the obtained results in terms of the amount of load. Figure 15 shows this process, for the ease of its use, the relationship governing the points has also been obtained. These changes can be used up to the dimensionless load equal to 45, and for larger loads, we have to use all the points of the interval, so that the interval from zero to the radius of the plate is divided into at least 7 equal parts and we use the obtained numbers in the equation machine. Table 4 summarizes the extracted relationships.

Advance Researches in Civil Engineering
ISSN: 2645-7229, Vol.5, No.1, pages: 70-85

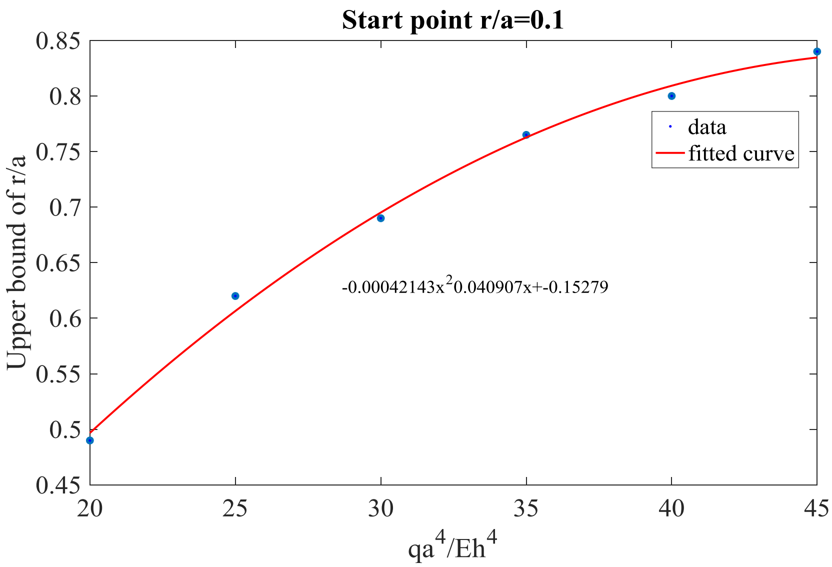


Figure 15. Characteristic of the last point according to loading.

Table 4. Summary of relationships for calculating division points according to loading.

qa^4 / Eh^4	Points	The equation	
3-11	2	-0.022x+0.7714	
11-16	3	0.2	$0.0094x^2$-0.22x+2.12
		0.3	$0.006x^2$-0.15x+1.65
		0.4	$0.0012x^2$-0.03x+0.96
		0.5	$-0.0072x^2$+0.17x-0.21
17-21	3	0.7	$0.0037x^2$-0.2x+2.7
		0.8	$0.0002x^2$-0.011x+0.54
		0.9	$-0.0071x^2$+0.29x-2.5
21-45	9	*Start 0.1	$-0.0004x^2$+0.04x-0.15
>45	9	1	

* The interval between 0.1 to the end of the interval into 6 equal parts are divided, 7 points are obtained. $x = {qa^4}/{Eh^4}$

5. Conclusion

The use of the approximate method of point collocation in solving the investigated problems led to acceptable results and formulated outputs for use in uniform loading values, so that the error rate in the calculation of the plate deflection was evaluated to be less than 2%. It was tried to control the way of selecting the points by reaching the functional formulas for the coordinates of the selected points and increase the advantage of using point collocation method.

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